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An efficient incompatible mode finite element for explicit dynamics

Problem Statement

Standard displacement finite element formulations exhibit different locking phenomena. The usage of incompatible mode (IM) and Enhanced Assumed strain (EAS) formulations eliminate locking, but require expensive computations on the element level at each time-step.

Solution

To counter this problem alternate formulation for methods of Incompatible modes is introduced which can help overcome the problem and to avoid the reduction of time step size due to high eigenfrequencies mass is assigned to the incompatible degrees of freedom.

Alternate IM element formulation for Explicit Dynamics

In this alternate IM element the incompatible degrees of freedom are not condensed out of the system show as

$$\mathbf{M}_D\ddot{\mathbf{u}}_n^c + \mathbf{f}^{int}(\mathbf{u}_n^c, \tilde{\mathbf{u}}_n) = \mathbf{f}_{ext}^n,$$
$$\tilde{\mathbf{M}}_D\ddot{\tilde{\mathbf{u}}} + \tilde{\mathbf{f}}(\mathbf{u}_n^c, \tilde{\mathbf{u}}_n) = \mathbf{0}.$$

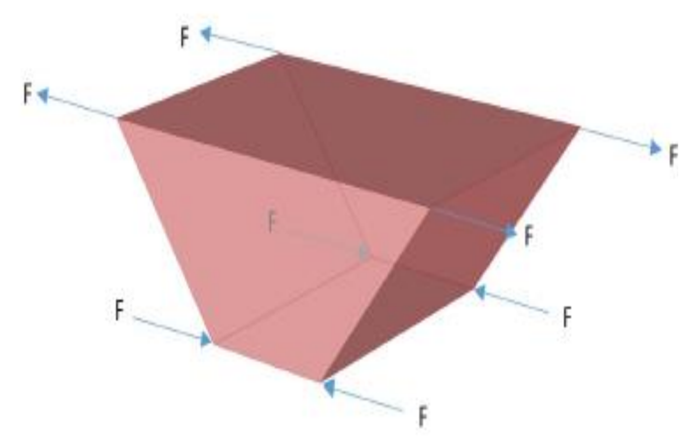
This removes the requirement of the linearization of the second equation which is required in EAS. The critical time step size for explicit dynamics is defined as

$$\Delta t_{crit} = \frac{2}{\omega_{max}}$$

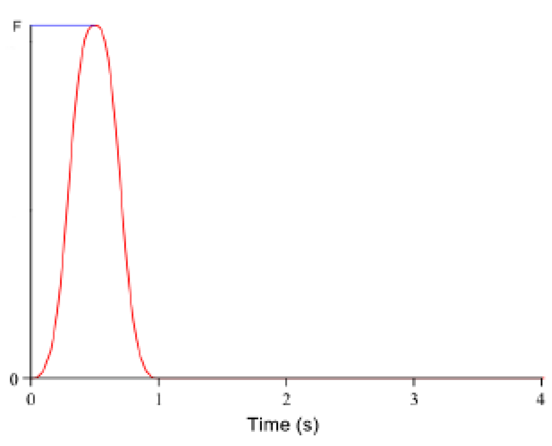
Inertia is assigned to these IM modes to avoid the increase of time step size due to high eigenfrequencies associated with them. This mass is defined by an artifical density $\tilde{\rho}$ which can be varied

$$\tilde{\rho}\int_V \tilde{\mathbf{N}}^T\tilde{\mathbf{N}}\,dV.$$

Single Brick element In Bending



Single Brick element in bending (Mode-A).



Single brick element when subjected to pure bending state behaves in between the enhanced assumed strain formulation and pure displacement formulation for varying values of $\tilde{\rho}$.

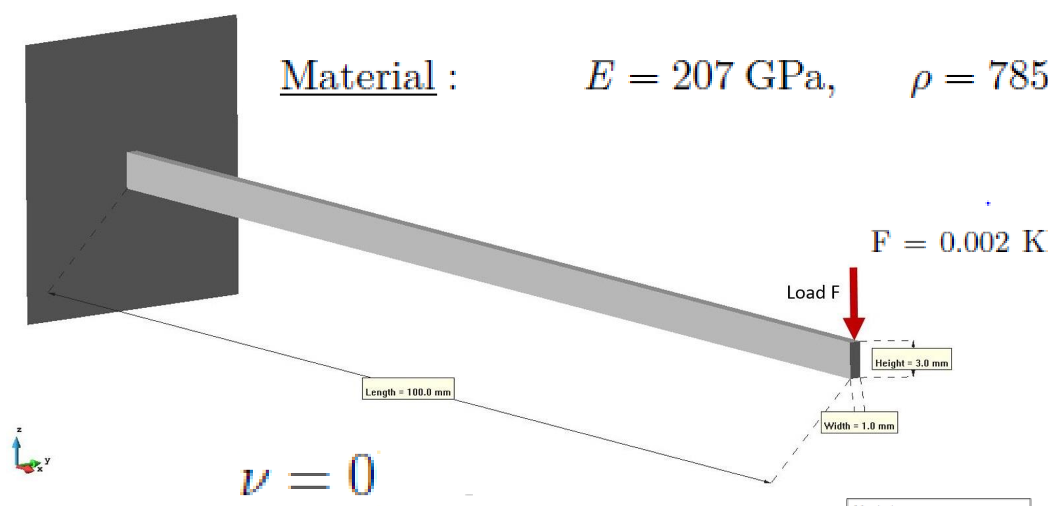
Decreasing the value of $\tilde{\rho}$ moves the formulation toward EAS while increasing it moves towards pure displacement formulaiton.

Numerical Examples

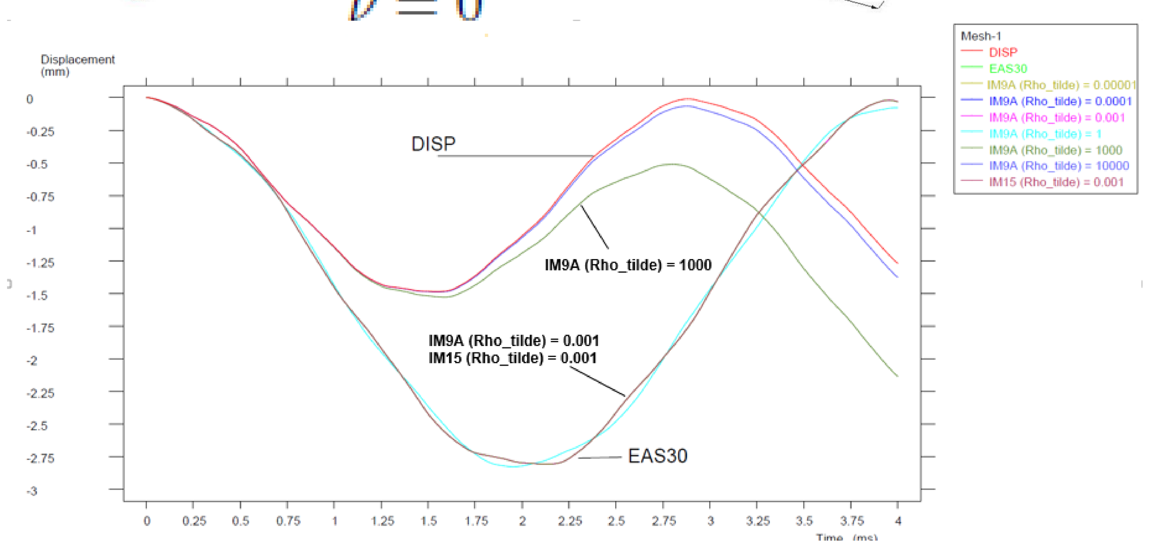
1.

$\underline{\text{Geometry}}$: Length $= 100.0$ mm, Width $= 1.0$ mm, Height $= 3.0$ mm

$\underline{\text{Material}}$: $E = 207$ GPa, $\rho = 7850$ x 10^{-9} kg/mm³



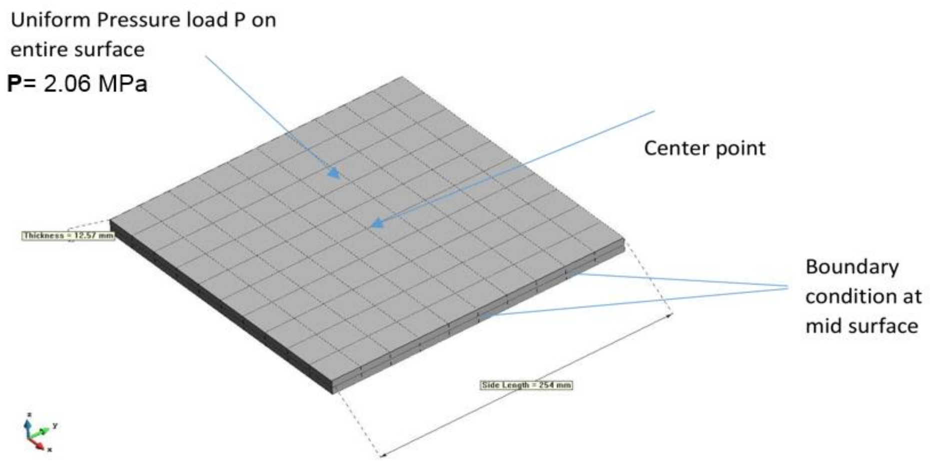
$\nu = 0$

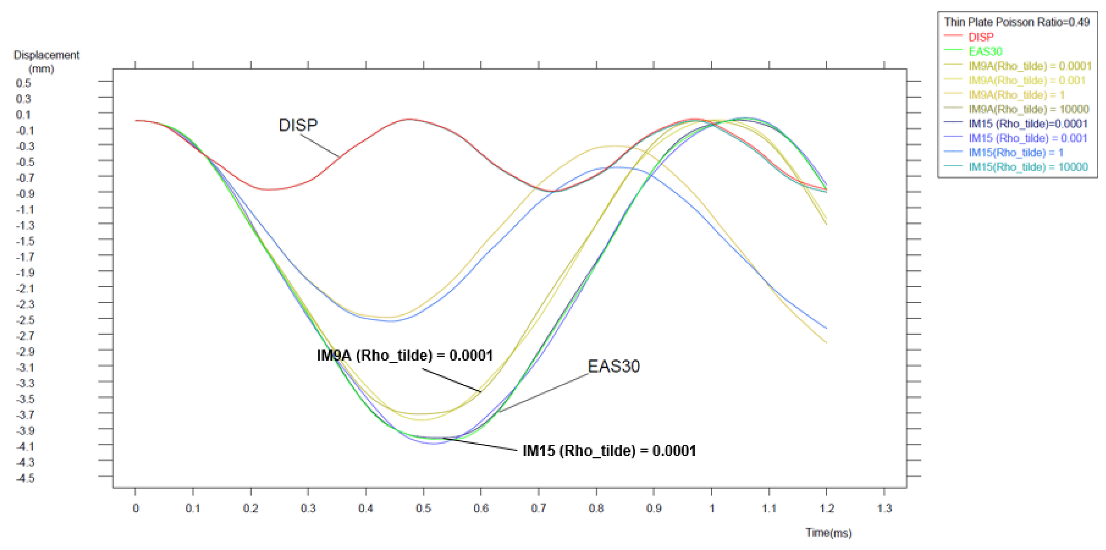


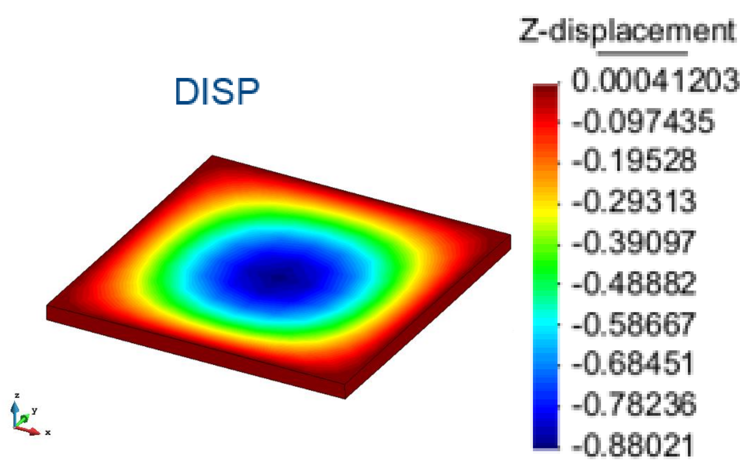
2.

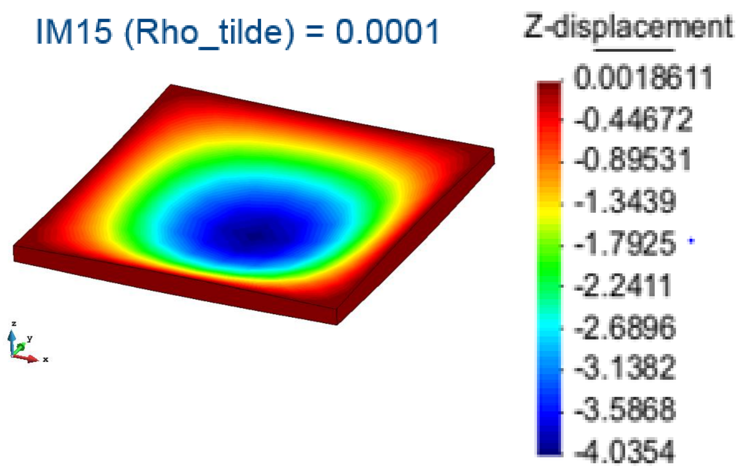
$\underline{\text{Geometry}}$: Length $= 254$ mm, Width $= 254$ mm, thickness $= 12.57$ mm

Material : $E = 69$ GPa, $\rho = 2760$ x 10^{-9} kg/mm³ $\nu = 0.49$









Literatur

H.A. Waqar; An efficient incompatible mode finite element for explicit dynamics, Master Thesis. IBB Stuttgart, 2017

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