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Mixed Finite Elements for Geometrically Nonlinear Problems

Motivation

Overstiff behavior, or “locking”, is one of the major problems with the standard displacement-based finite element method. Locking occurs when Poisson’s ratio $\nu \approx 0.5$ or thickness t ≈ 0.

In order to model structures more accurately, mixed finite element methods can be used. Using 1D, 2D, and 3D benchmark problems, this thesis compares the performance of two mixed finite element methods with the displacement-based method for geometrically nonlinear problems.

Finite Element Methods

Standard Method: Displacement-based FEM

- Based on the Principle of Virtual Work functional
- Displacement **u** is the primary variable.

$$\mathbf{K}\Delta\mathbf{d} = \mathbf{R}$$

Mixed FEM: Hybrid Stress Method

- Based on the Hellinger-Reissner functional
- Displacement **u** and stress **S** are primary variables
- In 2D, 5-parameter stress interpolation matrix (PS-5) is used.

$$\begin{bmatrix} -\mathbf{H} & \mathbf{G} \\ \mathbf{G}^{\mathrm{T}} & \mathbf{K}_{\mathrm{g}} \end{bmatrix} \begin{bmatrix} \Delta\beta \\ \Delta\mathbf{d} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{F}^{\mathrm{ext}} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{F}}^{\mathrm{int}} \\ \mathbf{F}^{\mathrm{int}} \end{bmatrix}$$

Mixed FEM: Enhanced Assumed Strain Method (EAS)

- Based on the Hu-Washizu functional
- Displacement **u** and assumed strain $\widetilde{E}$ are primary variables
- In 2D, 4- or 7-parameter strain interpolation matrix is used (EAS-4, EAS-7).

$$\begin{bmatrix} \mathbf{K}_{\mathrm{eu}} + \mathbf{K}_{\mathrm{g}} & \mathbf{L}^{\mathrm{T}} \\ \mathbf{L} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \Delta\boldsymbol{\alpha} \\ \Delta\mathbf{d} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{F}^{\mathrm{ext}} \end{bmatrix} - \begin{bmatrix} \tilde{\mathbf{F}}^{\mathrm{int}} \\ \mathbf{F}^{\mathrm{int}} \end{bmatrix}$$

Results

- Overall, the displacement-based method shows weaker performance in all benchmark problems.
- Even when the problem is locking-free, mixed elements require fewer iterations and obtain more consistent normal force values.
- Although the number of iterations may vary, in 2D examples PS-5 and EAS-7 elements and in 3D examples PS-18 and EAS-30 elements result in the identical displacement values.
- When the St. Venant-Kirchhoff material law is used, the hybrid stress method performs better than the EAS method. For other hyperelastic material models, the EAS method is recommended, due to the implementation challenges of the hybrid stress method.

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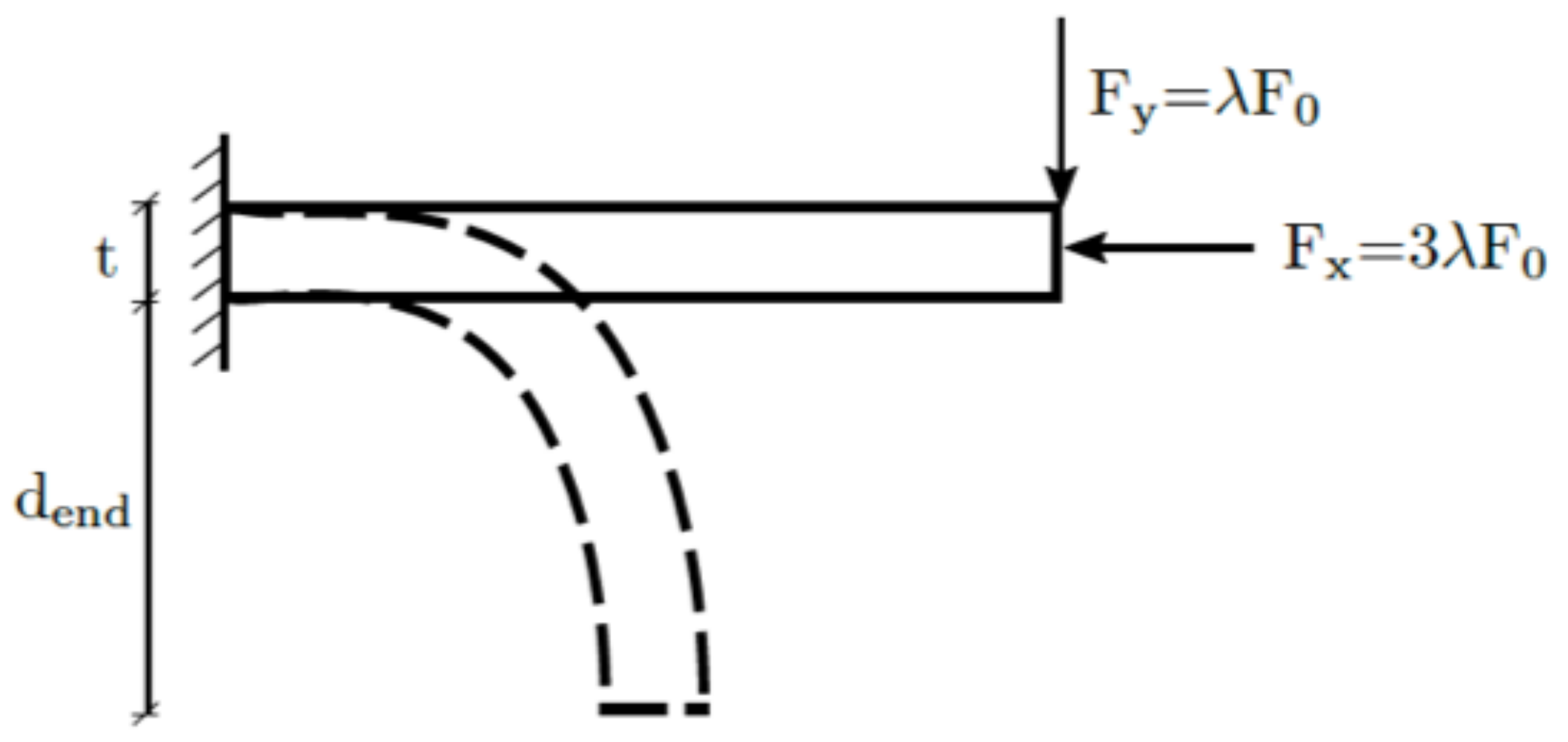
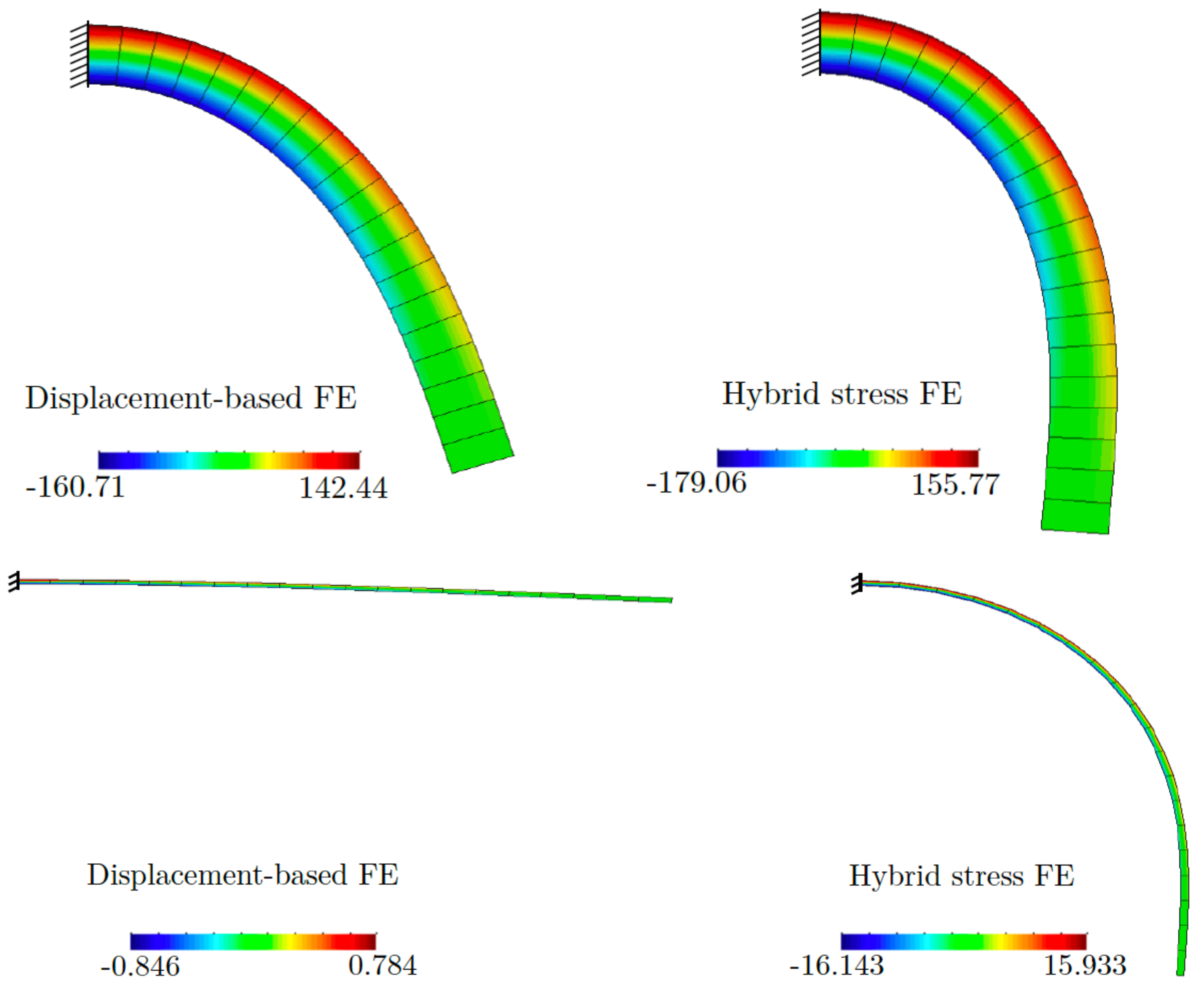
Numerical Example

Problem data:

- E=1000, ν=0.3
- L=1, t=1 and t=0.1
- St. Venant-Kichhoff material model
- F_x=F_y=-1, λ=t^3
- tol=10^{-7}



2D beam element with large displacement (Simo and Rifai, 1990). The finite element analysis is conducted for 20 elements with 10 load steps.



Deformed figures and the stress distributions (S_x) for thickness t=1 and t=0.1

References

- Simo, J.C.; Rifai, M.S. (1990). A class of mixed assumed strain methods and the method of incompatible modes. *International journal for numerical methods in engineering*
- Pian, T.H.H.; Sumihara, K. (1984) Rational approach for assumed stress finite elements. *International journal for numerical methods in engineering*

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