Universität Stuttgart
Fakultät für Bau- und Umweltingenieurwissenschaften

Chen Liang

Substructuring and Model Order Reduction

Motivation

Substructuring is mainly used in dynamics, so that also called as dynamic substructuring. The basic idea and thought is similar to finite element method, to divide a whole system into several smaller system, i.e. substructures, and then assemble substructures altogether. By dynamic analysis of substructures with large number of degrees of freedom, it may take prohibitive computational time during simulation. So that, model order reduction is necessary, to reduce the dimension of matrix and save run time of implementation. There are three model order reduction techniques are investigated in this thesis, namely static condensation, Guyan reduction and Craig-Bampton reduction.

Model Order Reduction Methods

Main concept of model order reduction (MOR) can be presented by following equation:

$$X_n = \begin{bmatrix} X_p \\ X_s \end{bmatrix} = TX_p$$

It means, all degree of freedom (dof) of system are condensed by preserved dofs, where p<<n. All dofs can be classified as two kinds, preserved and secondary dofs, according to MOR. The energy equation of system must be conservative, it follows:

$$E = \frac{1}{2} {X_n}^T K X_n = \frac{1}{2} {X_p}^T T^T K T X_p$$

So that, condensed stiffness- and mass- matrix is given. Due to transformation matrix, dimensions of matrix are already reduced.

$$K_{con} = T^T K_n T \qquad M_{con} = T^T M_n T$$

Guyan Reduction

Key points of each MOR techniques is transformation matrix,

$$T = \begin{bmatrix} I_{bb} \\ -{K_{ii}}^{-1} K_{ib} \end{bmatrix}$$

Craig-Bampton Reduction

It includes constrain mode and normal mode,

$$T = \begin{bmatrix} I_{bb} & o \\ \Phi^c & \Phi^N \end{bmatrix}$$

Conclusions

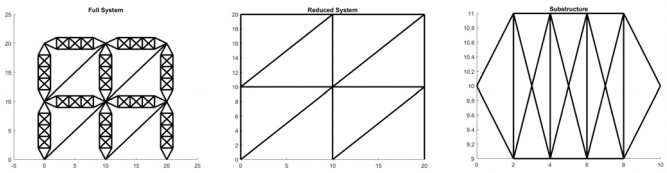
- Static condensation, exact results without loss any accuracy;
- Guyan reduction, only a few reasonable results are obtained, however, an efficient method;
- Craig-Bampton reduction, results with high accuracy are obtained, but more computational time required.

Supervisors:
Anton Tkachuk, Dr.-Ing.
Anika Strauß, M.Sc.
https://www.ibb.uni-stuttgart.de

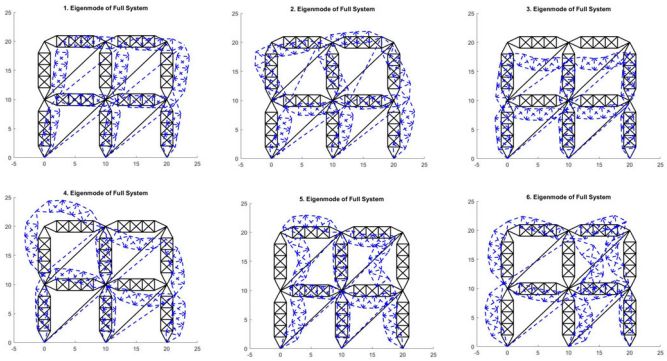
Numerical examples

Illustrative example structure

Implementation by Matlab, original system, reduced system and substructures are displayed below:



Eigenmodes of original system:



By Craig-Bampton reduction, the results from eigenproblem of reduced system own very nearly approximation in comparison to those from original system. Furthermore, local eigenmodes of substructures are obtained, while Guyan reduction doesn‘t have.

Literatur

Guyan, Robert: Reduction of stiffness and mass matrices. In: AIAA Journal, American Institute of Aeronautics and Astronautics (1965)

Roy Craig, Mervyn B.: Coupling of Substructures for Dynamic Analyses. In: AIAA Journal, American Institute of Aeronautics and Astronautics (1968)

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