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Locking-free Finite Elements in Finite Elasticity Problems

Motivation

Nowadays, finite element method (FEM) is the most widely used numerical method in civil engineering to analyze the response of structures under different conditions. Although many locking-free elements performs stably for linear problems, artificial numerical instabilities are still possibly caused in the finite deformation range. The aim of this thesis is to implementation and investigate performance of the selected Q1 element and EAS element in the finite element formulations in terms of locking and stability behaviour.

Nonlinear Finite Element Methods

Basic Equations of Continuum:

- Kinematics (geometrically non-linearity): $\boldsymbol{E} = \frac{1}{2}(\boldsymbol{F}^T\boldsymbol{F} - \boldsymbol{I})$
- Material (hyperelastic): $\boldsymbol{S} = \frac{\partial \Psi}{\partial \boldsymbol{E}}$
- Equilibrium (principle of virtual work):

$$\delta W = \underbrace{\int_B \boldsymbol{S} : \delta \boldsymbol{E}\, dV}_{\delta W_{int}} \underbrace{- \left(\int_B \rho_0 \bar{\boldsymbol{b}} \cdot \delta \boldsymbol{u} dV + \int_{\partial B} \bar{\boldsymbol{t}} \cdot \delta \boldsymbol{u} dA \right)}_{-\delta W_{ext}}$$

Stability Problem:

- State of equilibrium: $\boldsymbol{R} = \boldsymbol{F}_{int} - \boldsymbol{F}_{ext} = \boldsymbol{0}$
- Critical point of stability: $(\boldsymbol{K}_T - \omega \boldsymbol{I})\boldsymbol{\Phi} = \boldsymbol{0}$

Elements:

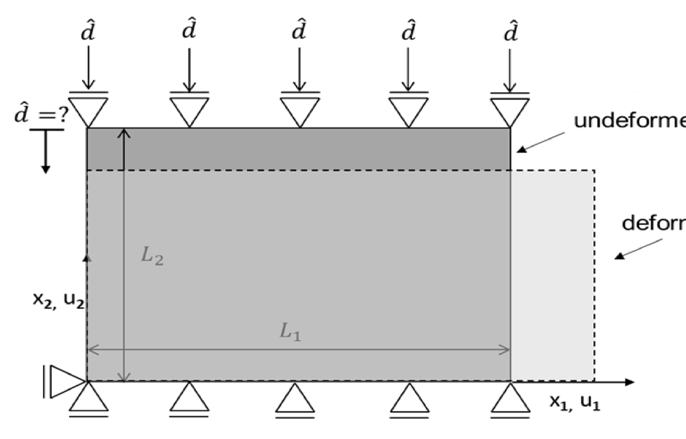
- Q1: $\boldsymbol{K}_T \delta \boldsymbol{d} = -\boldsymbol{R}$
- EAS: $\begin{bmatrix} \boldsymbol{K}_g + \boldsymbol{K}_e + \boldsymbol{K}_u & \boldsymbol{L}^T \\ \boldsymbol{L} & \boldsymbol{D} \end{bmatrix} \cdot \begin{bmatrix} \delta \boldsymbol{d} \\ \delta \boldsymbol{\alpha} \end{bmatrix} = \begin{bmatrix} \boldsymbol{F}_{ext} \\ \boldsymbol{0} \end{bmatrix} - \begin{bmatrix} \boldsymbol{R} \\ \tilde{\boldsymbol{R}} \end{bmatrix}$

Numerical Algorithms:

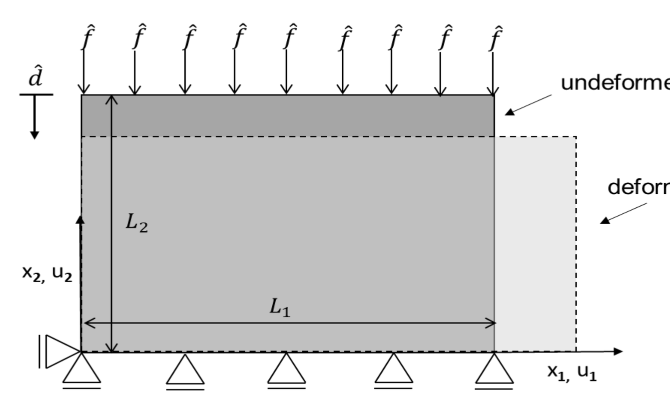
- Bisection method: $Delta_{lam} = \frac{1}{10^{j-1} \cdot num_{increments}}$
- Extended system:

$$\begin{bmatrix} \boldsymbol{K}_T & -\alpha \boldsymbol{I} & -\boldsymbol{F}_{ext} & -\boldsymbol{\Phi} \\ \frac{\partial \boldsymbol{K}_T}{\partial \boldsymbol{D}} \boldsymbol{\Phi} & \boldsymbol{K}_T & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0}^T & 2\boldsymbol{\Phi}^T & 0 & 0 \\ \boldsymbol{0}^T & \boldsymbol{F}_{ext}{}^T & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta \boldsymbol{D} \\ \Delta \boldsymbol{\Phi} \\ \Delta \lambda \\ \Delta \alpha \end{bmatrix} = - \begin{bmatrix} \boldsymbol{R}_1 \\ \boldsymbol{R}_2 \\ R_3 \\ R_4 \end{bmatrix}$$

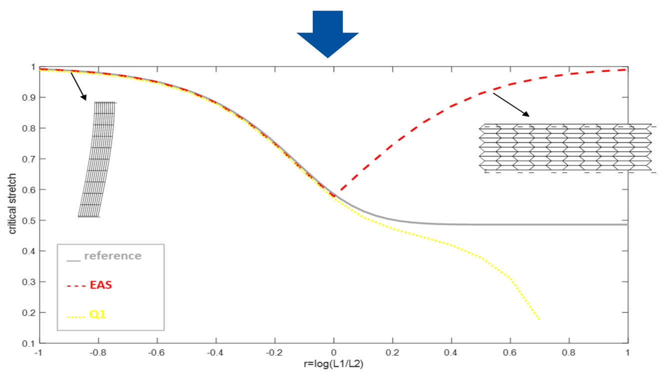
Numerical Example



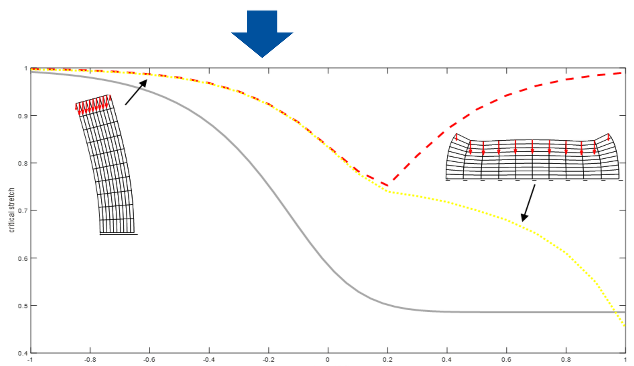
Model of displacement control



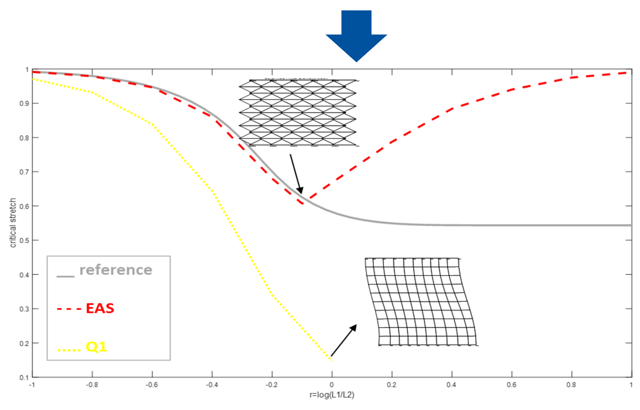
Model of load control



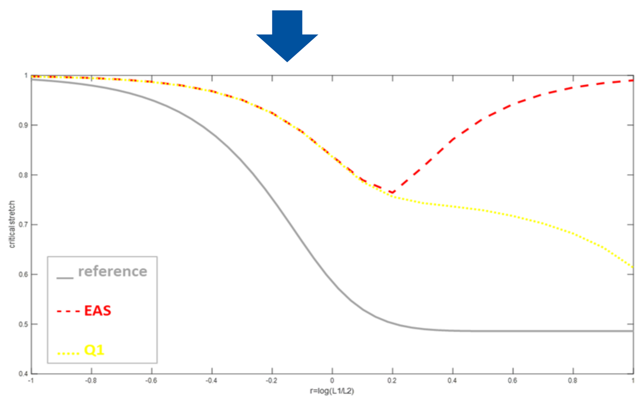
Stability Spectrum 10*10 with Bisection Method, $\nu = 0$



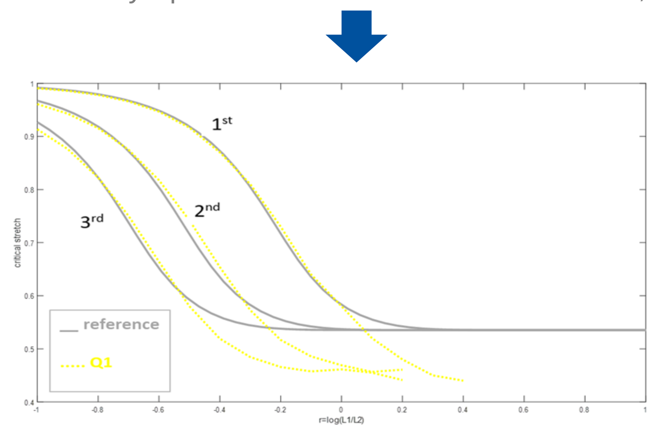
Stability Spectrum 10*10 with Extended System, $\nu = 0$



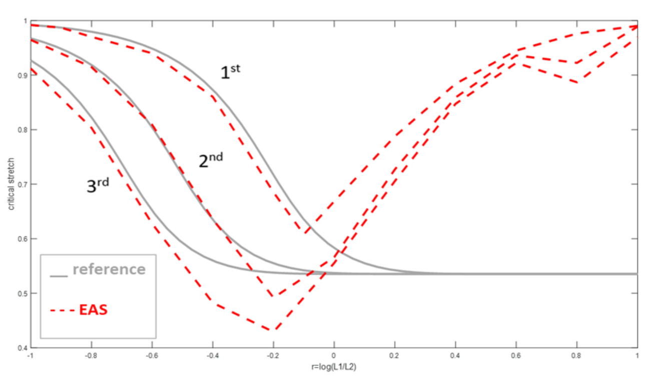
Stability Spectrum 10*10 with Bisection Method, $\nu = 0.499$



Stability Spectrum 16*16 with Extended System, $\nu = 0$



Stability Spectrum of Q1 16*16 with Bisection Method, $\nu = 0.45$



Stability Spectrum of EAS 16*16 with Bisection Method, $\nu = 0.45$

Conclusion

- In case of displacement control, the stability spectrums of both Q1 and EAS element are in good agreement with the analytical solution for a column - like model. Simulation with EAS elements is instable(hour-glassing effect) for a beam - like model. The mechanical property of the material plays a key role in the FEM analyze by using Q1 element.
- In case of load control, the simulated stability spectrum of Q1 and EAS elements are not in accordance with the analytical solution over the entire r range. Further investigations for this issue should be conducted to figure out the reason.

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https://www.ibb.uni-stuttgart.de

References

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