

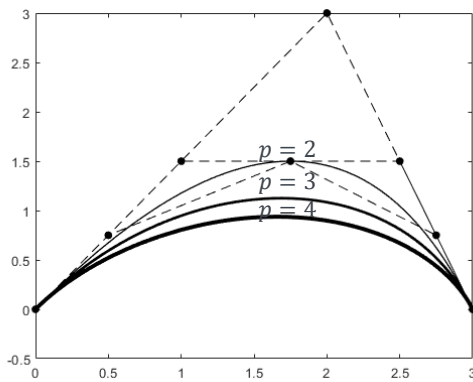
# Modal and transient structural analyses with NURBS solid finite elements using LS-Dyna

## Motivation

- NURBS(Non-uniform Rational B-Spline) basis functions possess the property of affine transformation such that it represents all rigid-body motions correctly and satisfies the patch tests.
- Unique properties of NURBS basis functions allow efficient and highly accurate approximations in structural analyses

## Unique Properties of NURBS

- Non-negativity :  $N_{i,p} \geq 0$
- $C^{p-k}$  Continuity : Spline basis functions can have higher continuity across the elements
- Variation Diminishing Property : No oscillation is observed in the higher-order approximation



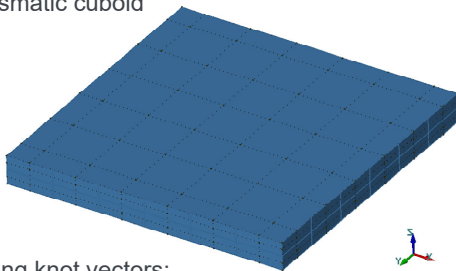
NURBS curves with the order(p) elevation

## Construction of NUBRS Solid

- Tensor product of univariate NURBS basis functions

$$(u, v, w) := \frac{N_{i,p}(u)M_{j,q}(v)L_{k,r}(w)h_{i,j,k}}{\sum_{i=1}^n \sum_{j=1}^m \sum_{k=1}^l N_{i,p}(u)M_{j,q}(v)L_{k,r}(w)h_{i,j,k}}$$

- E.g. Prismatic cuboid



Consisting knot vectors:

- $t(u) = [0 \ 0 \ 0 \ 0.5 \ 1 \ 1 \ 1]$
- $t(v) = [0 \ 0 \ 0 \ 0.16 \ 0.33 \ 0.50 \ 0.66 \ 0.83 \ 1 \ 1 \ 1]$
- $t(w) = [0 \ 0 \ 0 \ 0.16 \ 0.33 \ 0.50 \ 0.66 \ 0.83 \ 1 \ 1 \ 1]$

## Numerical Example

### Eigenvalue Analysis: Simply-supported Cuboid

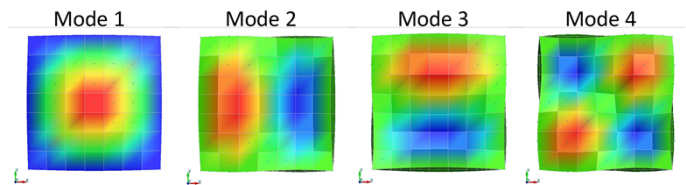
- Geometry: 10m square plate of thickness 1m, the element aspect ratio( $t/l_e$ ) of 0.26
- Material Properties: Young's modulus  $200 \times 10^9 \text{ N/m}^2$ , Poisson's ratio 0.3, Density  $8000 \text{ kg/m}^3$ ,

Results: (CMM :Consistent Mass, LMM: Lumped Mass)

- Rel. difference of first 4 eigenfrequencies

Element Type	Hexahedron			Tetrahedron		NURBS	
Order	1	1	2	1	2	2	3
Gp	1x1x1	2x2x1	2x2x2	1	4 or 5	2x2x2	3x3x3
Element Formulation	Elform 1	Elform 2	Elform 3	Elform 10	Elform 16	Knot Insertion	Standard k-refine.
Mass Matrix	CMM	CMM	LMM	CMM	CMM	CMM	CMM
No. of Nodes	324	324	324	324	1831	324	455
Rel. Difference	1	-3.44%	12.00%	0.42%	187.28%	-0.61%	0.25%
	2	1.06%	20.84%	1.48%	160.08%	0.51%	0.95%
	3	1.06%	20.84%	1.48%	160.08%	0.51%	0.95%
	4	0.71%	16.94%	0.43%	220.44%	1.42%	1.06%
CPU Time(sec)	1	1	2	1	2	1	2
Memory(Mbytes)	0.38	0.39	4.24	0.30	3.01	0.89	2.76

- All the formulations contain the first 4 reference eigenmodes



- Comparing with the conventional finite element system, the NURBS system shows computationally more efficient and better accuracy
- Further experiments also showed that the NURBS element has strong resistance to the severely poor aspect ratio ( $t/l_e \approx 0$ )

## Literature

- Hughes, Thomas JR and Cottrell, John A and Bazilevs, Yuri, Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, Computer methods in applied mechanics and engineering