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Computational Implementation of The Ultimate Load Method

Motivation

Linear programming is one of the most mature and widely used methods in mathematical planning, and its solution methods include simplex method, primary matrix method, interior point method, saddle point method and decomposition method of special structure matrix. In this paper, the limit analysis problem is transformed into a linear programming problem based on the simplex method for solving the ultimate load of a plane rigid frame. This method only uses the information of the original structure to calculate and does not need the information of the stiffness of the structure, the uniform load is also easy to handle, and it is convenient to implement the program.

Linear programming method

Based on the lower limit theorem, each element has to satisfy the equilibrium condition and the internal force limitation condition. In the case of no uniform load, the maximum absolute value of bending moment in the element must be at the two ends of the element, so only the moment at the two ends of the element can be controlled by applying the internal force limitation condition.

$$|\mathbf{M}_i| \le \mathbf{M}_{pl} \quad or \quad -\mathbf{M}_{pl} \le \mathbf{M}_i \le \mathbf{M}_{pl} \quad (i=1,2)$$

By assembling the element equations, the following global equilibrium equation is obtained:

$$\mathbf{H}\mathbf{F}_X^* = \mathbf{P}_J - \mathbf{F}_{pl}$$

With the global equilibrium equation, the problem of solving the ultimate load based on the lower bound theorem can be formulated as a linear programming problem as follows:

$$\begin{cases} min(-\lambda) \\ s.t. \begin{bmatrix} \mathbf{a}_1 & \mathbf{H} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{I} \end{bmatrix} \begin{bmatrix} \lambda \\ \mathbf{F}_X^* \\ \mathbf{W} \end{bmatrix} = \begin{bmatrix} -\mathbf{F}_{pl} \\ 2\mathbf{M}_{pl} \end{bmatrix} \end{cases}$$

$$p \ge 0, \quad \mathbf{F}_X^* \ge 0, \quad \mathbf{W} \ge 0$$

Conclusion

The linear programming method is a method to find the ultimate value of a linear function under certain structural constraints. By transforming the structural constraints by translational transformations and introducing relaxation variables, the ultimate load problem can be transformed into a standard linear programming problem, and the ultimate load of the structure can be obtained by the simplex method. The linear programming method is computationally efficient and can directly derive the final state of the structure, and it can handle the uniform load better.

Supervision:
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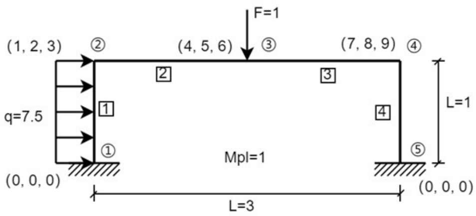
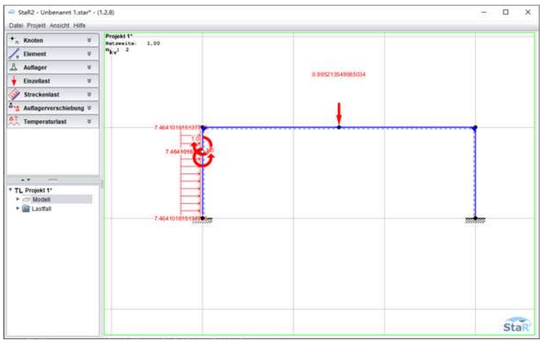
https://www.ibb.uni-stuttgart.de

Illustrative Examples





$$\mathbf{x} = [0.995 \quad 0 \quad 2 \quad 1.732 \quad 0.268 \quad 0.120 \quad 1.880 \quad 0 \quad 2 \quad 2 \quad 2 \quad 0 \quad 0.268 \quad 1.732 \quad 1.880 \quad 0.2 \quad 2 \quad 0 \quad 0]$$

Literature

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