Universität Stuttgart
Fakultät für Bau- und Umweltingenieurwissenschaften

Uygar Kovanci

Investigation and Mitigation of Volumetric Locking for Higher-Order Tetrahedral Elements

Motivation

In Finite Element Analysis (FEA), tetrahedral elements are very attractive to use in 3-D mesh generation due to automatized mesh algorithms. However, the FEA with these elements encounter a big challenge: Volumetric locking. The studies showed that volumetric locking can occur even in fine meshes. In contrast, higher convergence rates in terms of displacement results are observed when higher-order displacement-based elements are used. However, the spurious modes lead stress results to oscillate. Therefore, a new higher-order tetrahedral element formulation is required to eliminate these modes by utilizing robust and open-source C++ FEM library Ikarus.

Modified Tetrahedral Element

Element stiffness matrix (**k**) is separated into constant and hourglass components $\nu \neq 0$ (correct material law)

$$\mathbf{k} = \mathbf{k}_c + \mathbf{k}_h.$$

Thus, spurious modes that are dependent on Poisson's ratio are stored in the hourglass part. The same decomposition is applied to the stiffness matrix where $\nu = 0$ (false material law)

$$\mathbf{k}^0 = \mathbf{k}_c^0 + \mathbf{k}_h^0.$$

If the Poisson's ratio is zero, the element has no spurious mode. So, the hourglass stiffness from the correct material law is replaced by the hourglass stiffness from the false material law

$$\begin{aligned}\overline{\mathbf{k}} &= \mathbf{k}_c + \mathbf{k}_h^0 \\ &= \underbrace{\sum_{gp=1}^{4} \mathbf{B}^T\mathbf{C}^0\mathbf{B}|\mathbf{J}|}_{\mathbf{k}^0} + \underbrace{\left(\mathbf{B}^T(\mathbf{C}-\mathbf{C}^0)\mathbf{B}|\mathbf{J}|\right)_\xi}_{\mathbf{k}_c-\mathbf{k}_c^0}.\end{aligned}$$

Eventually, the modification affects the formulation of the stress matrix as

$$\begin{aligned}\overline{\boldsymbol{\sigma}} &= \boldsymbol{\sigma}_c + \boldsymbol{\sigma}_h^0 \\ &= \underbrace{\mathbf{C}^0\mathbf{B}\mathbf{d}\Big|_{\xi_{\mathrm{GP}}}}_{\boldsymbol{\sigma}^0} + \underbrace{(\mathbf{C}-\mathbf{C}^0)\mathbf{B}\mathbf{d}\Big|_{\xi}}_{\boldsymbol{\sigma}_c-\boldsymbol{\sigma}_c^0}\end{aligned}$$

Numerical Comparison (Thick Pipe Test)

Second-order standard tetrahedral element (TET10) and modified tetrahedral element (MTET10) are compared in terms of radial displacement, normal stresses (σ) and triaxiality (η).

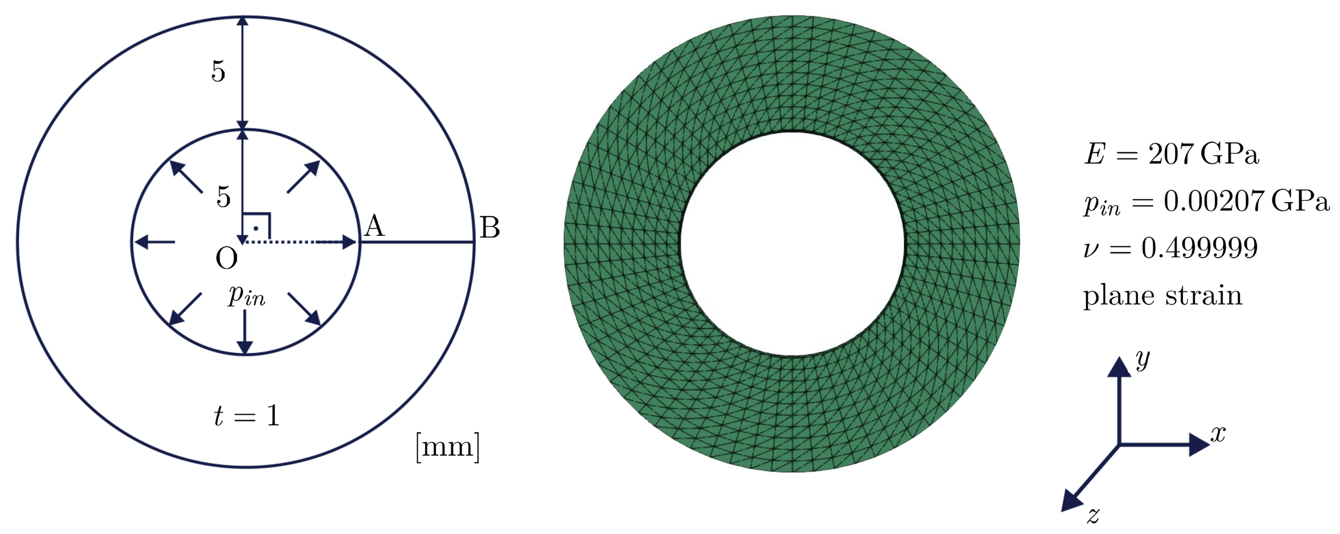

Figure 1: Dimensions and mesh configuration of thick pipe model (DOFs: 6336)

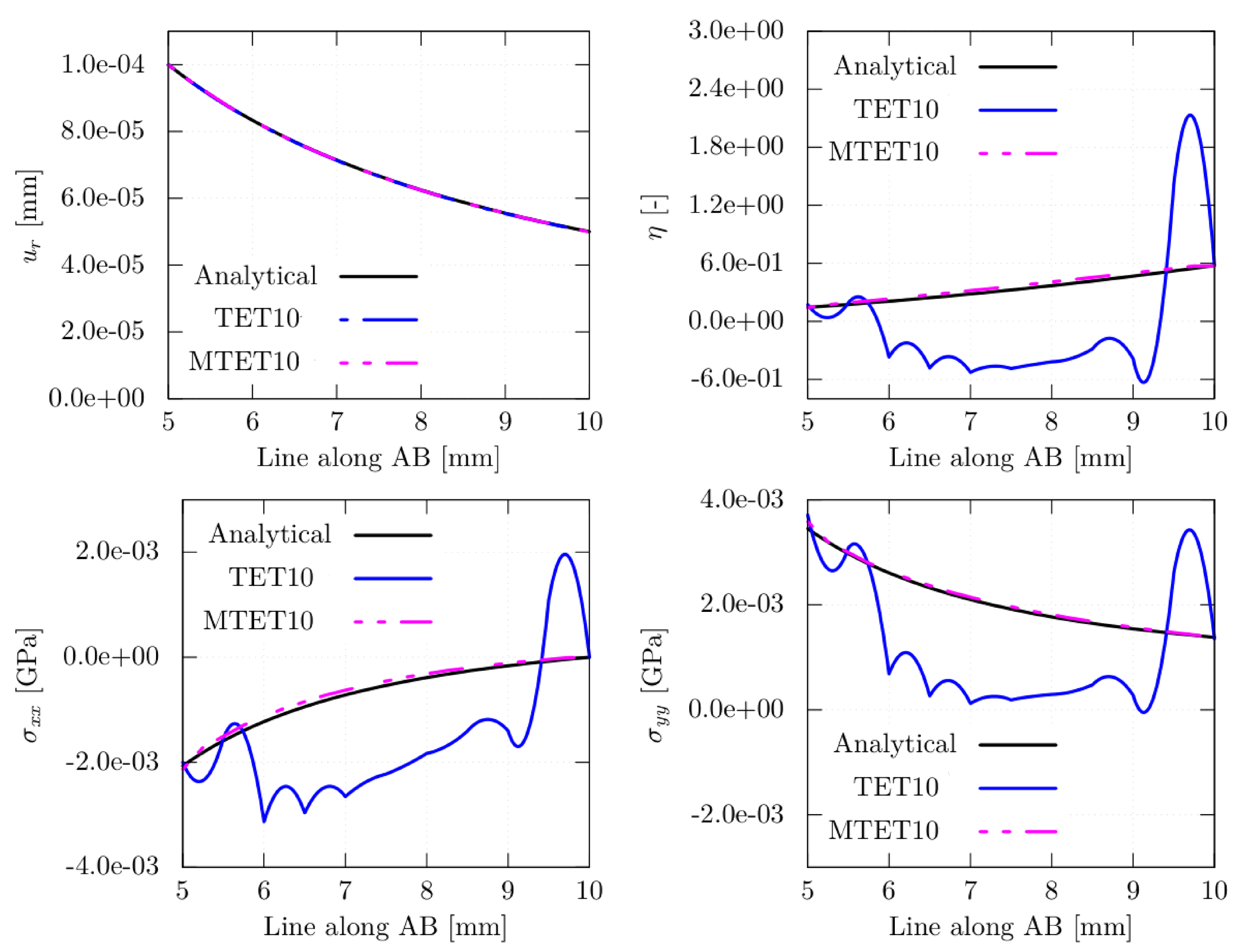

Figure 2: Radial displacement, stresses in x- and y-directions, triaxiality results along AB line

Conclusion

The modified TET10 (MTET10) is locking-free and oscillatory behavior of stress results due to locking is eliminated. The results of the thick pipe test are almost aligning with the analytical results even in very coarse meshes. Thus, in engineering and scientific applications of nearly incompressible materials, one can opt for MTET10 element to obtain precise results.

Literature

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Supervisors:
Tarun Kumar Mitruka Vinod Kumar Mitruka, MSc.
Maximilian Schilling, MSc.
https://www.ibb.uni-stuttgart.de

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