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Investigations on the Optimization of the Redundancy Distribution in Truss Structures

Motivation

Understanding the redundancy distribution in structures reveals hidden resilience. It identifies critical elements whose failure would significantly compromise the safety of the structure. Optimizing redundancy distribution becomes crucial because excess capacity can be strategically redistributed to fortify the structure against unexpected element failure. This not only increases the robustness but also optimizes the material usage.

Theory

The calculation of the redundancy matrix is based on the principle of elastostatics.

$$\mathbf{R} = \mathbf{I} - \mathbf{A}(\mathbf{A}^{\mathrm{T}}\mathbf{C}\mathbf{A})^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{C}$$

$\mathbf{C}$: Member stiffness matrix

$\mathbf{A}$: Compatibility matrix

The matrix $\mathbf{C}$ contains the stiffness properties of the members. The compatibility matrix $\mathbf{A}$ is derived based on the geometric composition of a structure. Changes to the stiffness distribution or the geometric composition of a structure lead to alterations in the redundancy matrix. The aim of the changes is an even redundancy distribution.

Adaptations of the Stiffness Distribution:

The following equation is set up for every member of a structure. The repeated application of the formula leads to the member stiffness distribution that results in an even redundancy distribution.

$$C_i^{opt} = R_i \, C_i / R_i^{opt}$$

C_i: Member stiffness

R_i: Member redundancy

Adaptations of the Structure's Geometry:

The redundancy of a member is determined by the constraint of the surrounding system on that specific member. To change the constraint on a particular member at a specific node, the attention is directed towards adjusting the angles between the opposing members. Increasing the angle between opposing members reduces the constraint on the individual member, resulting in an increased redundancy.

Conclusion

To optimize the redundancy distribution in truss structures, the initial phase involves adjusting the structure's geometry to balance constraints. To achieve complete optimization, the last step is to modify the stiffness of the members.

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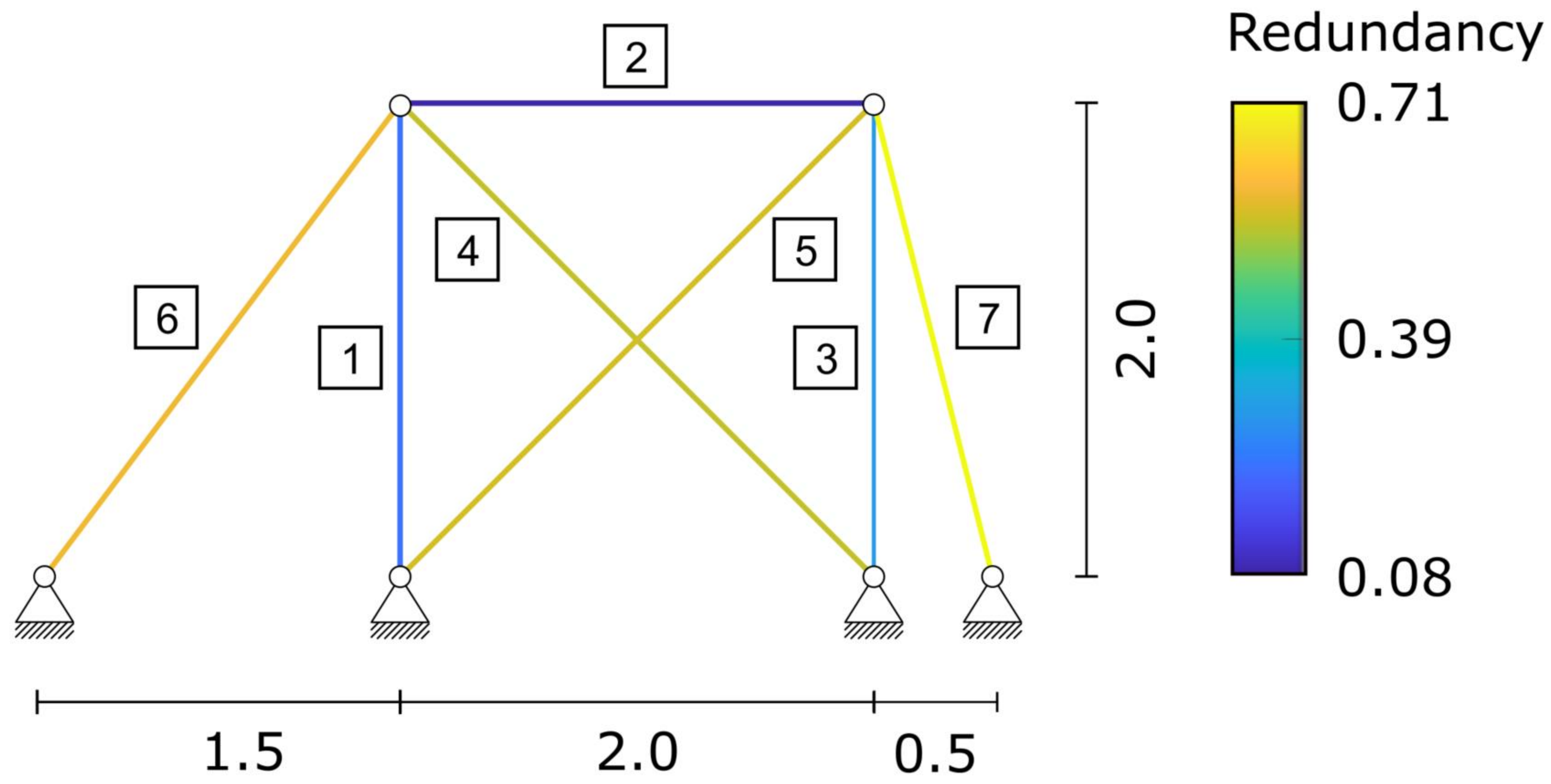
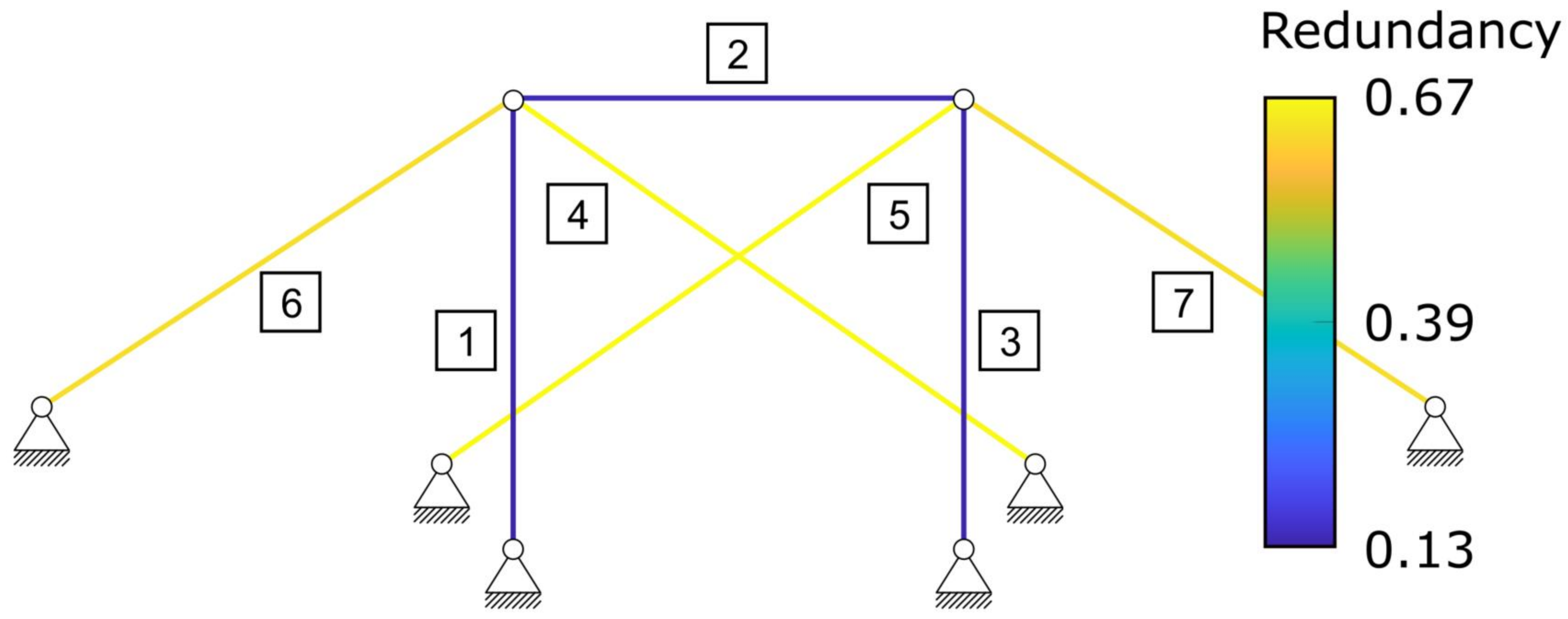
Illustrative Example



The initial member stiffness ratio between members 1-7 is:

1 : 1 : 1 : 0.2 : 0.2 : 0.3 : 0.3

1. The redundancy of member 2 is particularly low. To increase it, the angle between members 1 and 6, as well as between 1 and 4, is increased. The changes are mirrored. The redundancy of member 2 could be increased by 80%.



2. The member stiffnesses are modified for an even redundancy distribution. The final stiffness ratio between members 1-7 is:

1 : 1.1 : 1 : 1.2 : 1.2 : 1.2 : 1.2

Literature

Von Scheven., M; Ramm, E.; Bischoff, M.: Quantification of the redundancy distribution in truss and beam structures. International Journal of Solids and Structures 213, 41-49.(2021)

Baustatik und Baudynamik