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Comparison of the Mixed-Displacement method with alternative unlocking schemes

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Motivation

The Finite Element Method (FEM) is widely used in structural mechanics to solve complex continuous problems by transforming them into finite elements. However, it faces the issue of "locking“, where parasitic strains and stresses cause artificial stiffness, affecting accuracy. Various methods like Selective Reduced Integration (SRI), Assumed Natural Strain (ANS), Enhanced Assumed Strain (EAS), and Discrete Strain Gap (DSG) have been developed to mitigate locking. Despite these, locking remains intrinsically to the underlying differential equations. The Mixed-Displacement (MD) method offers a solution, which is prior to discretization schemes, using equal-order interpolations for locking-free results. Therefore, MD method needs to be investigated and compared its performance against the mentioned unlocking schemes.

Theoretical uniqueness of MD method

Mathematical formulation:

- Hellinger-Reissner principle:

$$\Pi_{\mathrm{HR}}(\mathbf{u},\bar{\mathbf{u}}) = \int_{\Omega} \left(\frac{1}{2}\mathbf{E}_{\bar{\mathrm{u}}}^{\mathrm{T}}\mathbf{C}\mathbf{E}_{\bar{\mathrm{u}}} - \mathbf{E}_{\mathrm{u}}^{\mathrm{T}}\mathbf{C}\mathbf{E}_{\bar{\mathrm{u}}}\right) \mathrm{d}\Omega + \int_{\Omega} \mathbf{u}^{\mathrm{T}}\mathbf{b}\,\mathrm{d}\Omega + \int_{\Gamma_\sigma} \mathbf{u}^{\mathrm{T}}\hat{\mathbf{t}}\,\mathrm{d}\Gamma_\sigma \rightarrow \mathrm{stat.}$$

- An independent strain field $\boldsymbol{E}$:

$$\mathbf{E} =: \mathbf{E}_{\bar{\mathrm{u}}} = \bar{\mathcal{L}}\bar{\mathbf{u}}$$

- Idea of MD method: to link the strains $\boldsymbol{E}$ to additional degrees of freedom $\bar{u}$ through a specific linear differential operator $\bar{\mathcal{L}}$.
- For 2D linear elasticity:

$$\mathbf{E}_{\mathrm{u}} \approxeq \boldsymbol{\varepsilon}_{\mathrm{u}} = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial}{\partial x} & 0 \\ 0 & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} \end{bmatrix}}_{\mathcal{L}} \underbrace{\begin{bmatrix} \mathrm{u_x} \\ \mathrm{u_y} \end{bmatrix}}_{\mathbf{u}} \longrightarrow$$

$$\mathbf{E}_{\bar{\mathrm{u}}} \approxeq \boldsymbol{\varepsilon}_{\bar{\mathrm{u}}} = \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ 2\varepsilon_{xy} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial^2}{\partial x \partial y} \end{bmatrix}}_{\bar{\mathcal{L}}} \underbrace{\begin{bmatrix} \bar{\mathrm{u}}_{\mathrm{x}} \\ \bar{\mathrm{u}}_{\mathrm{y}} \\ \bar{\mathrm{u}}_{xy} \end{bmatrix}}_{\bar{\mathbf{u}}}$$

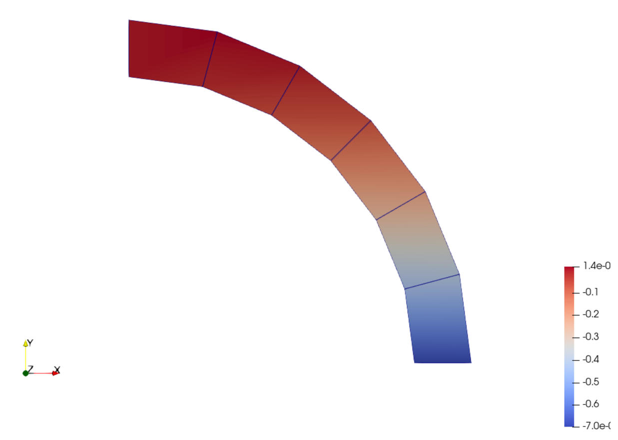
- For Reissner-Mindlin plate model:

$$\mathbf{E}_{\mathrm{u}} = \underbrace{\begin{bmatrix} \frac{\partial}{\partial x} & 0 & 1 \\ \frac{\partial}{\partial y} & -1 & 0 \end{bmatrix}}_{\mathcal{L}} \underbrace{\begin{bmatrix} \nu \\ \varphi_{\mathrm{x}} \\ \varphi_{\mathrm{x}} \end{bmatrix}}_{\mathbf{u}} \longrightarrow$$

$$\mathbf{E}_{\bar{\mathrm{u}}} = \underbrace{\begin{bmatrix} \frac{\partial}{\partial x} & \\ & \frac{\partial}{\partial y} \end{bmatrix}}_{\bar{\mathcal{L}}} \underbrace{\begin{bmatrix} \bar{\nu}_{xs} \\ \bar{\nu}_{ys} \end{bmatrix}}_{\bar{\mathbf{u}}}$$

Numerical examples

Curved beam:



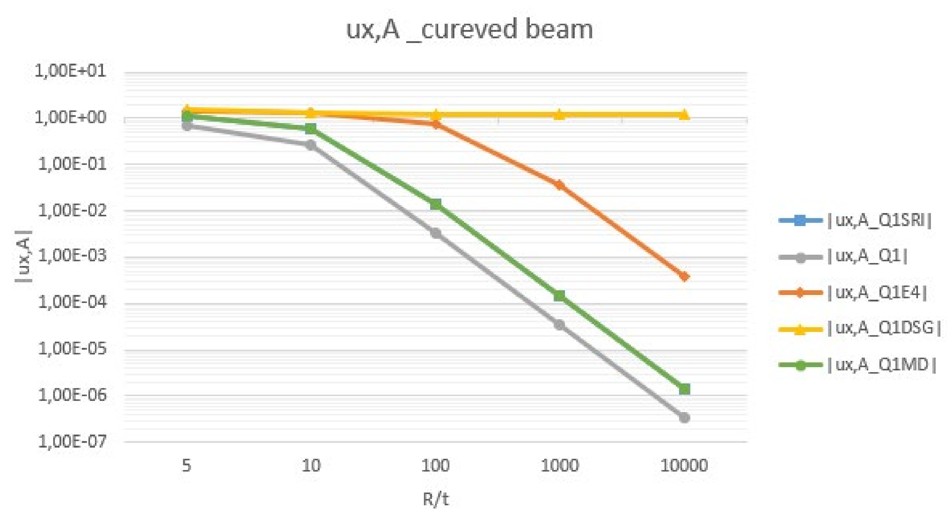


Fig.1 contour plot: displacement of Q1 element

Fig.2 displacement of vertex inside vs. slenderness

Simply supported plate:

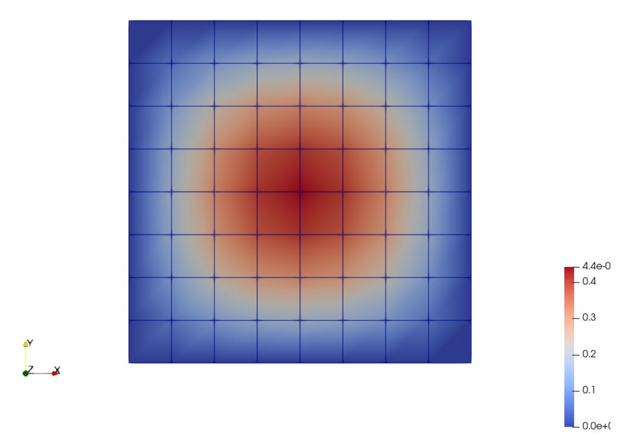


Fig.3 contour plot: midpoint-displacement of Q1ANS for the mesh with refinement =3

Fig.4 midpoint displacement vs. number of element

Summary

in 2D solid elements:

- MD method performs very similar to SRI scheme
- effective in avoiding shear locking
- potential to be extended to arbitrary discretization schemes

in plate elements:

- MD method performs very similar to ANS scheme
- effective in avoiding shear locking

References

T. K. M. V. K. Mitruka1 and M. Bischoff, “The mixed displacement method to avoid shear locking in problems in elasticity,” submitted

S. Bieber, B. Oesterle, E. Ramm, and M. Bischoff, “A variational method to avoid locking—independent of the discretization scheme,” International Journal for Numerical Methods in Engineering, vol. 114, no. 8, pp. 801–827, 2018.

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