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The Mixed Displacement method and Polygonal Plate Finite Elements

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Motivation

Polygonal elements have shown to be advantageous in many aspects of the finite element technology in structural engineering. They provide:

- High flexibility in meshing arbitrary geometries.
- Local mesh refinement is easily applicable.
- No restriction on the number of edges per element.

For Reissner-Mindlin plates, purely displacement-based formulations are prone to shear locking phenomena. This issue is treated with the mixed displacement method, which is:

- A formulation that returns intrinsically locking free results by equal order interpolation.
- Locking free, independent of the discretization scheme.

Scaled Boundary Finite Elements

A scaled boundary element is partitioned into sectors by connecting all boundary vertices with the scaling center *O*. The local coordinate system is defined by the scaling coordinate ξ and the circumferential coordinate η, where

$\eta \in [-1{,}1]$ and $\xi \in [0{,}1]$.

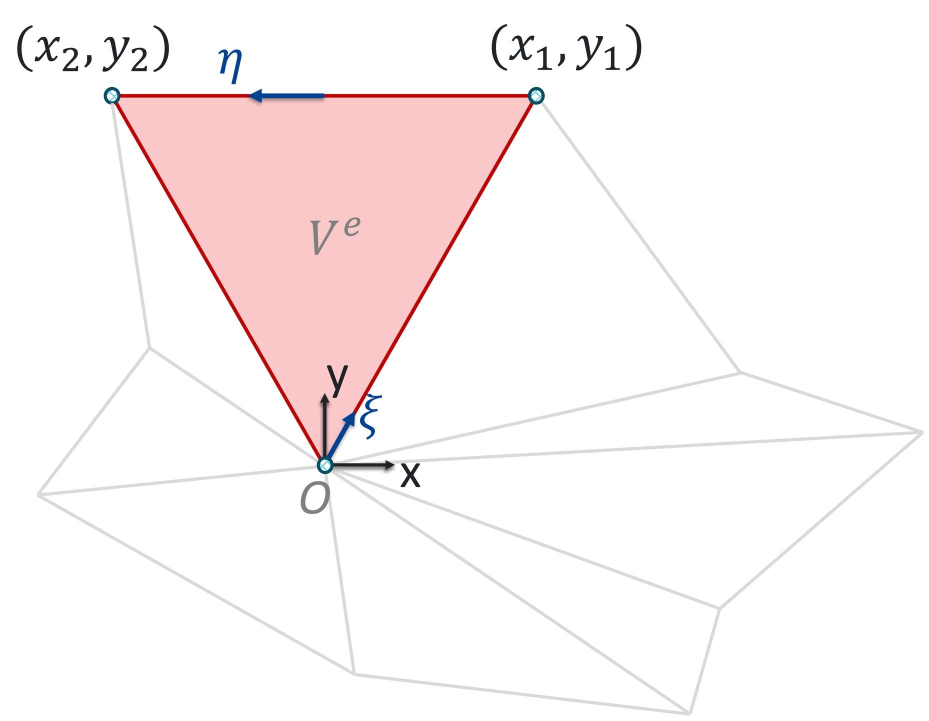

Fig. 1: Scaled boundary element with local coordinate system partitioned into sectors V^e

Mixed Displacement Method

The mixed displacement principle can be expressed as a multifield functional in terms of the displacement field $\mathbf{u}$ and an additional field $\bar{\mathbf{u}}$ that represents the integral of the shear strains. These additional terms remove geometrical locking effects intrinsically. The internal part of the functional which is derived from the Hellinger-Reissner functional reads:

$$\Pi^{\text{int}}_{\text{HR}}(\mathbf{u},\bar{\mathbf{u}}) = \int_\Omega \left(\frac{1}{2}\,\boldsymbol{\varepsilon}_{\bar{\mathbf{u}}}^T\,\mathbf{C}\,\boldsymbol{\varepsilon}_{\bar{\mathbf{u}}} - \boldsymbol{\varepsilon}_{\mathbf{u}}^T\,\mathbf{C}\,\boldsymbol{\varepsilon}_{\bar{\mathbf{u}}}\right) \mathrm{d}\Omega$$

The constraints on unstructured polygonal meshes can be enforced via a penalty method. The first variation of the complete functional reads:

$$\delta\Pi^{\text{int}}_{\text{HR}}(\mathbf{u},\bar{\mathbf{u}}) = \int_\Omega \Big(\delta\boldsymbol{\varepsilon}_{\bar{\mathbf{u}}}^T\mathbf{C}\,\boldsymbol{\varepsilon}_{\bar{\mathbf{u}}} - \delta\boldsymbol{\varepsilon}_{\mathbf{u}}^T\mathbf{C}\,\boldsymbol{\varepsilon}_{\bar{\mathbf{u}}} - \boldsymbol{\varepsilon}_{\mathbf{u}}^T\mathbf{C}\,\delta\boldsymbol{\varepsilon}_{\bar{\mathbf{u}}} - \epsilon_p\delta\boldsymbol{\varepsilon}_p^T\mathbf{C}\,\boldsymbol{\varepsilon}_p\Big)\,\mathrm{d}\Omega$$

Supervision

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Displacements of the Simply Supported Plate Problem

Polygonal meshes can be generated through centroidal Voronoi tessellation (CVT). To create well-shaped, uniformly sized elements, the Lloyd algorithm is an essential component of the CVT procedure, significantly impacting the solution.

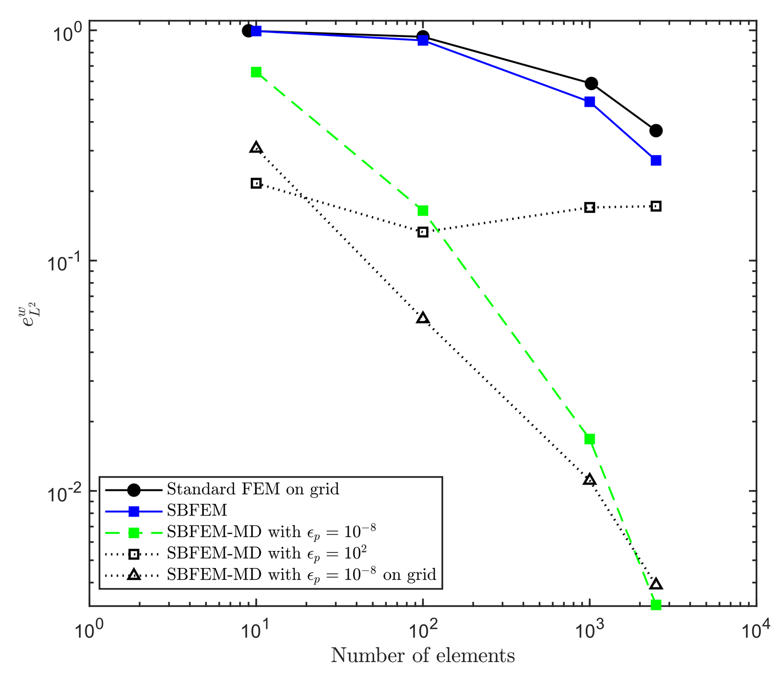

Fig. 2: L²-norm of relative displacement errors of the plate domain $e^{w}_{L^2}$ against the number of elements with a slenderness of 100

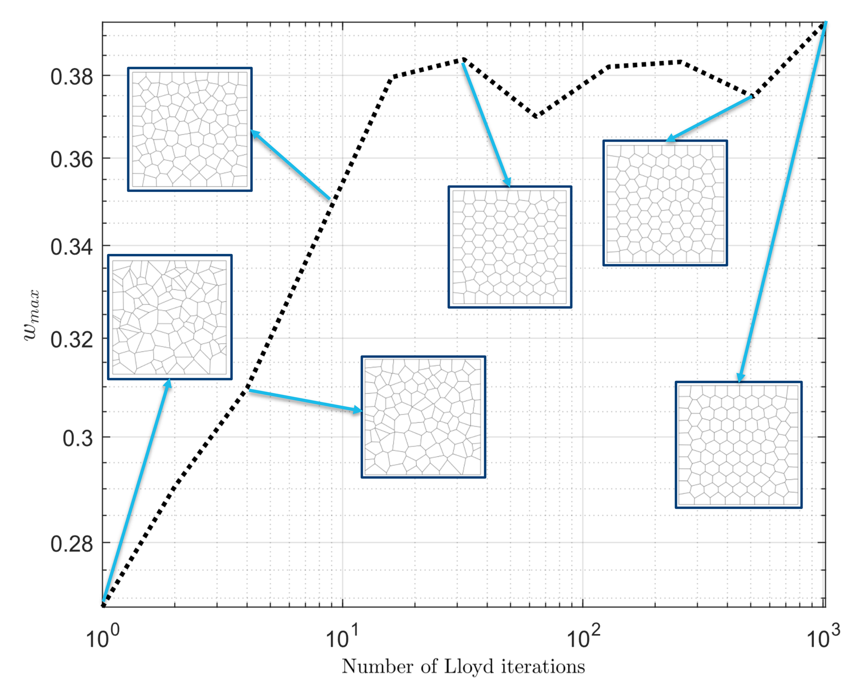

Fig. 3: Maximum displacement in the center of the plate against the number of Lloyd iterations with a slenderness of 100

Results

- The mixed displacement method with scaled boundary parametrization returns far better results for both structured and unstructured Voronoi meshes compared to the standard, purely displacement based finite element formulation.
- The uniformity and smoothness of the polygonal mesh has a significant impact on the results.
- Observed shear force oscillations suggest that the current implementation has not resolved all locking effects entirely.
- It is generally possible to place the scaling center on arbitrary locations inside and outside the respective element for a pure scaled boundary finite element formulation.

Literature

Bieber S, Oesterle B, Ramm E, Bischoff M. A variational method to avoid locking—independent of the discretization scheme. Int J Numer Methods Eng. 2018

Bjorn Sauren, Simon Klarmann, Leif Kobbelt, Sven Klinkel, A mixed polygonal finite element formulation for nearly-incompressible finite elasticity, Computer Methods in Applied Mechanics and Engineering, Volume 403, Part A, 2023

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