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Implementation and Comparison of Different Methods for the Exact Calculation of Critical Points

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Motivation

In the context of nonlinear analysis of thin-walled structures, failure due to buckling rather than material yielding is not only possible but is often the primary cause of structural collapse. Pre-buckling methods, although simple and inexpensive, may substantially overestimate the critical load. Computing a complete load-displacement path up to the critical load is computationally expensive and does not determine the critical points exactly. For this reason, a range of methods for the exact computation of critical points has been developed in the literature, most notably the minimal augmentation method, the null vector method, and a variant of the bisection method.

Methods

Compared with bisection, direct methods make use of extended systems and utilize constraint equations to converge directly towards a critical point.

The minimal augmentation method extends the equilibrium equations with the constraint of a zero eigenvalue, that is, singularity of the tangent stiffness matrix.

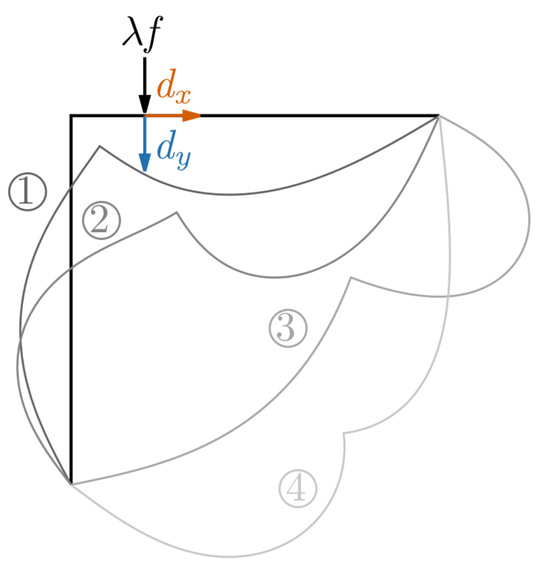
$$\mathbf{r}(\mathbf{d}, \lambda) = \begin{bmatrix} \mathbf{f}_{\text{int}}(\mathbf{d}) - \lambda \mathbf{f}_{\text{ext}} \\ g_1(\mathbf{d}) \end{bmatrix} = \mathbf{0}$$

The null vector method uses an equivalent formulation: at a critical point, a deformation in the direction of the buckling vector requires zero force.

$$\mathbf{r}(\mathbf{d}, \lambda) = \begin{bmatrix} \mathbf{f}_{\text{int}}(\mathbf{d}) - \lambda \mathbf{f}_{\text{ext}} \\ \mathbf{K}_{\text{T}}(\mathbf{d})\boldsymbol{\varphi} \\ l(\boldsymbol{\varphi}) \end{bmatrix} = \mathbf{0}$$

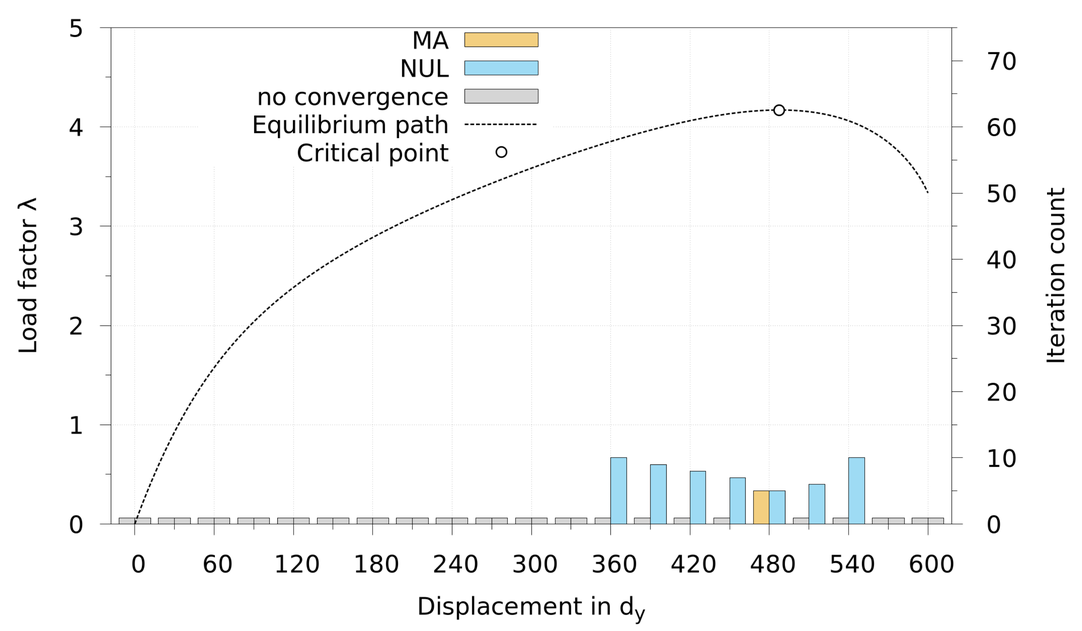
Both methods are solved using a Newton-Raphson algorithm. To enhance convergence, either of two improvements can be implemented. The first, named the hybrid approach, replaces some iterations of the direct methods with equilibrium iterations to stabilize convergence. The second uses a line search algorithm to compute the optimal step size within each iteration.

The methods and their enhancements are tested using the Lee frame, which experiences snap-through under loading.

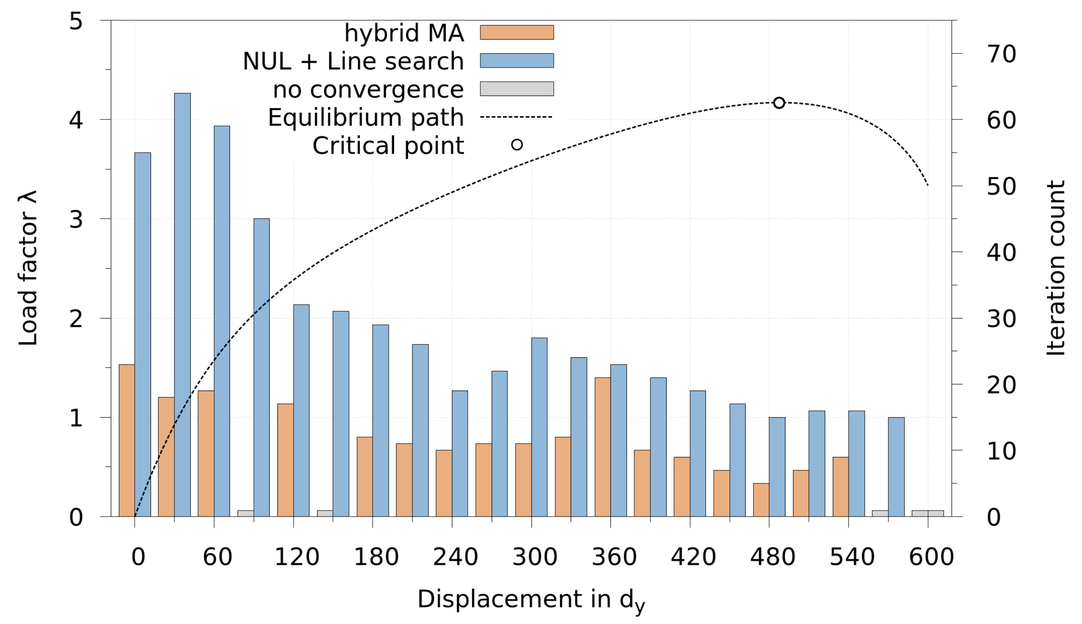


Results

Both unmodified methods are started from the undeformed configuration and 20 configurations along the load displacement path. Their iteration count, if successful, is displayed in the following plots:



As seen in the next plot, both modified methods are able to converge from less deformed configurations. Here, the minimal augmetation method using the hybrid approach and the null vector method using line search are displayed.



References

Wriggers et al., 1988 , “A quadratically convergent procedure for the calculation of stability points in finite element analysis”

Battini et al., 2003, “Improved minimal augmentation procedure for the direct computation of critical points”

Betreuung:

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