Avoiding Volumetric Locking in Ten-Node Tetrahedral Finite Elements for Hyperelasticity

Jonathan George Kuriakose

Universität Stuttgart
Fakultät für Bau- und Umweltingenieurwissenschaften

Motivation

The 10-node tetrahedral element is popular in Finite Elements due to the ease of tetrahedral mesh creation using automatic mesh generation algorithms. It is also free of shear locking, unlike the 4-node tetrahedral element. However, it still suffers from volumetric locking. Lo and Ling [1] proposed a method to treat volumetric locking; however, this method is only applicable to linear elastic materials. This method is extended for use in hyperelastic materials, and the resulting formulations are implemented and tested in LS-DYNA.

Modified Tetrahedral Element

The original formulation consists of modifying the element stiffness matrix. To extend it to hyperelasticity, the modification is applied to the internal force vector instead. The force vector is split into its constant and higher order part,

$$\mathbf{f}^{\mathrm{int}} = \mathbf{f}_{\mathrm{c}}^{\mathrm{int}} + \mathbf{f}_{\mathrm{h}}^{\mathrm{int}}.$$

The higher order part must be modified to remove volumetric locking. To do that, $\mathbf{f}^{\mathrm{int},0}$ is introduced, where the material parameters are modified to be locking-free, where the Poisson's ratio is 0. Then, a similar split is done as the original force vector,

$$\mathbf{f}^{\mathrm{int},0} = \mathbf{f}_{\mathrm{c}}^{\mathrm{int},\mathbf{0}} + \mathbf{f}_{\mathrm{h}}^{\mathrm{int},\mathbf{0}}.$$

The new modified internal force vector takes the original force vector and replaces $\mathbf{f}_{\mathrm{h}}^{\mathrm{int}}$ with $\mathbf{f}_{\mathrm{h}}^{\mathrm{int},\mathbf{0}}$,

$$\bar{\mathbf{f}}^{\mathrm{int}} = \mathbf{f}_{\mathrm{c}}^{\mathrm{int}} + \mathbf{f}^{\mathrm{int},0} - \mathbf{f}_{\mathrm{c}}^{\mathrm{int},0}.$$

$\mathbf{f}_{\mathrm{c}}^{\mathrm{int}}$ and $\mathbf{f}_{\mathrm{c}}^{\mathrm{int},0}$ are calculated with reduced order integration (numerical integration order $= 1$),

$$\mathbf{f}_{\mathrm{c}}^{\mathrm{int}} = [\mathbf{B}^{\mathrm{T}}\{\boldsymbol{\sigma}\}]_{\xi_{\mathrm{c}}}\left[\left|\mathbf{J}_{\xi}\right|\right]_{\xi_{\mathrm{c}}}(1),$$

$$\mathbf{f}_{\mathrm{c}}^{\mathrm{int},0} = [\mathbf{B}^{\mathrm{T}}\{\boldsymbol{\sigma}^{0}\}]_{\xi_{\mathrm{c}}}\left[\left|\mathbf{J}_{\xi}\right|\right]_{\xi_{\mathrm{c}}}(1).$$

$\mathbf{f}^{\mathrm{int},0}$ is calculated using full integration,

$$\mathbf{f}^{\mathrm{int},0} = \sum_{\mathrm{Q}=1}^{\mathrm{n_{GP}}} [\mathbf{B}^{\mathrm{T}}\{\boldsymbol{\sigma}^{0}\}]_{\xi_{\mathrm{Q}}}\left[\left|\mathbf{J}_{\xi}\right|\right]_{\xi_{\mathrm{Q}}} w_{\mathrm{Q}}\,.$$

$\boldsymbol{\sigma}$ and $\boldsymbol{\sigma}^{0}$ are given by the constitutive relation for a hyperelastic material model. For a compressible neo-Hookean model, this is given by,

$$\boldsymbol{\sigma} = J^{-1}(\lambda \ln(J)\,\mathbf{I} + \mu(\mathbf{F}\mathbf{F}^{\mathrm{T}} - \mathbf{I}))$$
$$\boldsymbol{\sigma}^{0} = J^{-1}(\lambda^{0}\ln(J)\,\mathbf{I} + \mu^{0}(\mathbf{F}\mathbf{F}^{\mathrm{T}} - \mathbf{I}))$$

λ^{0} is always 0, but μ^{0} may change depending on which material parameter is held constant. If the shear modulus is kept constant (LLM-TET10), $\mu^{0} = \mu$. If the Young's modulus is kept constant (UKM-TET10), $\mu^{0} = \mu(1 + \upsilon)$.

Due to the modified internal force vector, the post processing stress must also be modified. It is modified using a similar procedure to get the post-processing stress at an integration point,

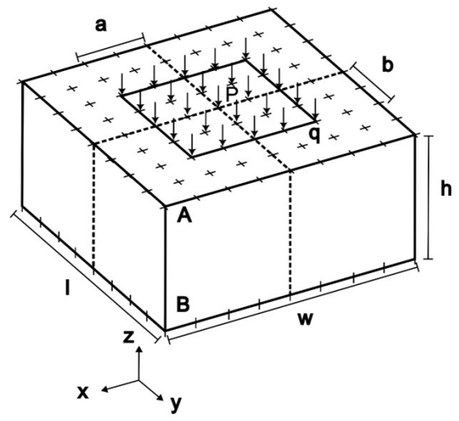
$$[\bar{\boldsymbol{\sigma}}]_{\xi_{\mathrm{Q}}} = ([\boldsymbol{\sigma}]_{c} - [\boldsymbol{\sigma}^{0}]_{c}) + [\boldsymbol{\sigma}^{0}]_{\xi_{\mathrm{Q}}}$$

Supervisor:

Tolga Usta, M.Sc.

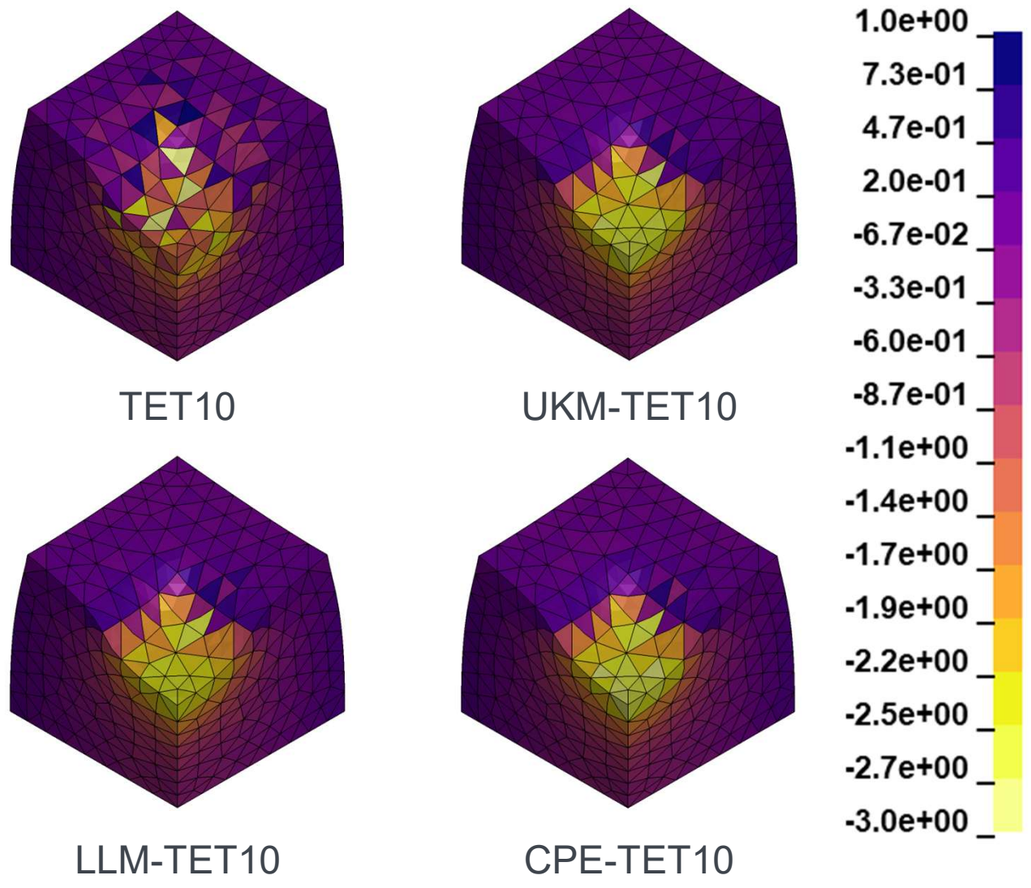
LS-DYNA Benchmark: Incompressible Block Under Constant Partial Load

The problem setup is described by the figure below.



$$\mathrm{E} = 4.829\ \mathrm{MPa}$$
$$\nu = 0.4984$$
$$q = 3.0\ \mathrm{MPa}$$
$$h = 50\mathrm{mm}$$
$$w = l = 100\mathrm{mm}$$
$$a = b = 25\mathrm{mm}$$

Symmetry is used to simulate only a quarter of the problem. The pressure field contour plots are compared between TET10, UKM-TET10, LLM-TET10 and CPE-TET10 (Cosserat Point Element).



TET10 | UKM-TET10

LLM-TET10 | CPE-TET10

The modified formulations show reduced volumetric locking, while being computationally cheaper than the CPE-TET10 element, which is known to deal with volumetric locking well.

Literature

[1] S. Lo and C. Ling, "Improvement on the 10-node tetrahedral element for three-dimensional problems," *Computer Methods in Applied Mechanics and Engineering*, vol. 189, no. 3, pp. 961–974, 2000. doi: https://doi.org/10.1016/S0045-7825(99)00410-7

Baustatik und Baudynamik