Model Order Reduction for Different Discretizations

Universität Stuttgart
Institute for Structural Mechanics

Nina He

1.Overview

Snapshot POD can reduce the cost of repeated finite-element simulations, but only after the displacement fields are written in a common reference format.
This poster shows how a common reference structure makes FEM results with different meshes, geometries, or topologies comparable before POD is applied.

2. Motivation and research question

- Structural optimization produces families of FE models with varying meshes, geometries and topologies.
- Raw displacement vectors cannot be stacked safely: entries may describe different nodes, locations, components or absent parts.
- The central question is how to define a common reference structure before snapshot POD is applied.

3.Workflow

A reference structure fixes the common meaning of the snapshot entries before POD is computed.

1. FEM solution
high-fidelity displacement field

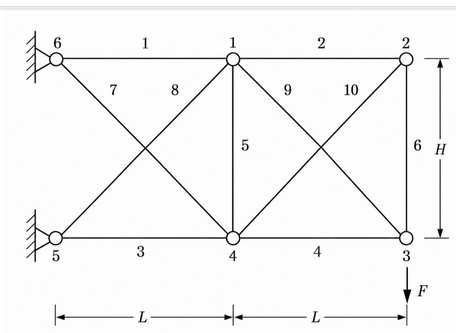
2. Reference representation
common nodes / DOFs or evaluation points

3. Snapshot matrix
columns with the same meaning

4. POD + test error
compact basis and reconstruction check

4. Benchmark 1: topology-varying truss

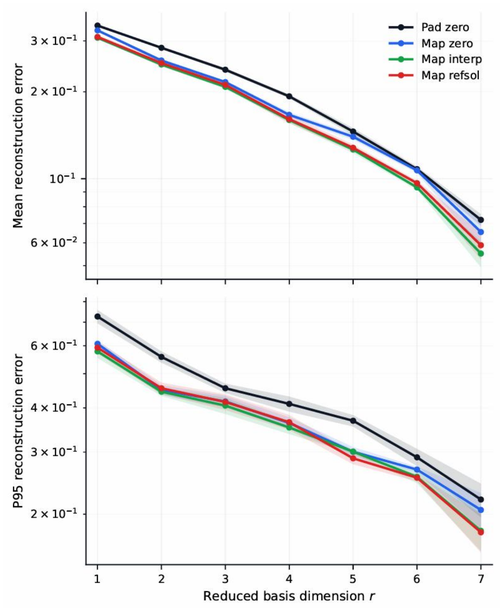
Topology variation means that members or nodes may be present in one sample but absent in another.



Parent truss used as the reference structure.

Compared rules

- **pad_zero** — same length only
- **map_zero** — same DOF meaning
- **map_interp** — fill absent DOFs by interpolation
- **map_refsol** — fill absent DOFs from a reference solution

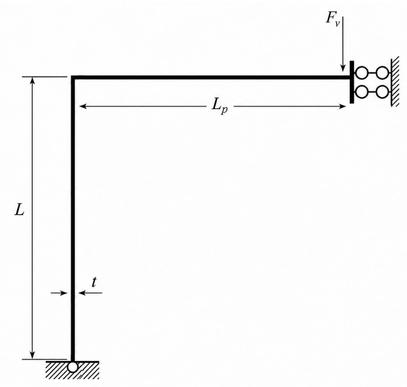


Mapping-based rules consistently improve over the length-only baseline.
The exact ranking still depends on the completion rule and the chosen evaluation norm.

Take-away: Equal length is not enough. DOF meaning matters.

5.Benchmark 2:lee frame

Mesh variation changes the FEM grid; geometry variation changes the beam length and solution domain.
Reference structure: a meshed reference Lee frame.
Here, the reference defines both DOF meaning and field-evaluation positions.



Case 1: mesh variation

Padding-based baselines remain near order-one transfer errors.
transfer_fe and transfer_gl2 both reduce the mean transfer error to 0.0793.
Practical choice in this benchmark: transfer_fe, because it is much cheaper (3291 ms vs. 86547 ms).

Selected comparison

Method	Mean error	Time (ms)
transfer_fe	0.0793	3291
transfer_gl2	0.0793	86547

Case 2: geometry variation

Three pipelines are compared: A follows the geometry; B and C-ref keep the reference nodes fixed.

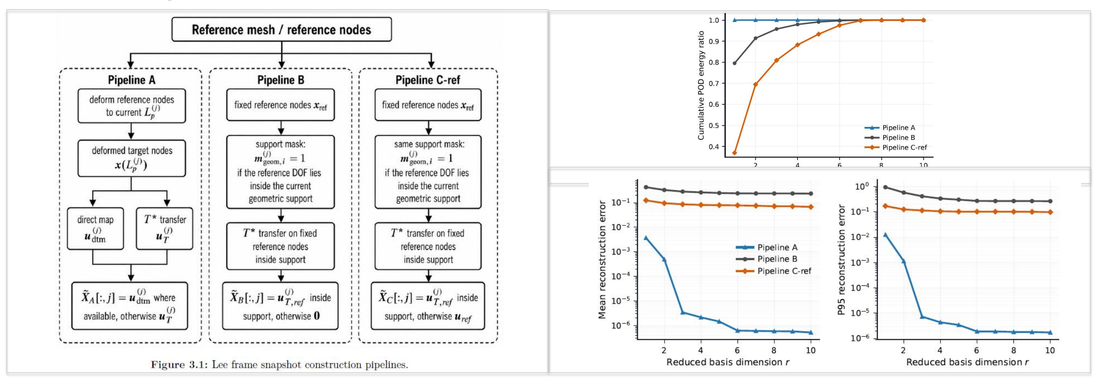


Figure 3.1: Lee frame snapshot construction pipelines.

Pipeline A is much more compact and reduces mean and P95 errors by several orders of magnitude.

6.Main Findings

POD for different discretizations is first a problem of constructing a meaningful snapshot matrix.

Equal length is insufficient; a reference structure is required.
Transfer and completion define the snapshot matrix.
POD energy alone is insufficient; reconstruction error must be checked.

7.Reference

[1] He, N. Model Order Reduction for Different Discretizations. Master Thesis, University of Stuttgart, 2026.
[2] Quarteroni, A., Manzoni, A., Negri, F. Reduced Basis Methods for Partial Differential Equations. Springer, 2016.
[3] Hesthaven, J. S., Rozza, G., Stamm, B. Certified Reduced Basis Methods for Parametrized Partial Differential Equations. Springer, 2016.
[4] Farrell, P., Maddison, J. Conservative interpolation between volume meshes by local Galerkin projection. CMAME, 2011.
[5] Taddei, T., Zhang, L. A discretize-then-map approach for the treatment of parameterized geometries in model order reduction. Adv. Comput. Math., 2021.

Betreuung:

Tarun Kumar Mitruka Vinod Kumar Mitruka, M.Sc.;
Anika Strauss, M.Sc.

Baustatik und Baudynamik