

# Buckling under Contact Constraints as a Source of Scatter in Car Crash Simulations

Anton Tkachuk<sup>1</sup>, Manfred Bischoff<sup>1</sup>

<sup>1</sup>Institute of Structural Mechanics, University of Stuttgart, Germany  
E-mail: tkachuk@ibb.uni-stuttgart.de, bischoff@ibb.uni-stuttgart.de

**Keywords:** contact, buckling, crash simulation, sensitivity.

The quality of car crash analysis improved dramatically in the past decade. The simulations include contact, large deformations and non-linear material laws, failure and spot-weld models. However, noisy response of such simulation is a hindrance to full-scale structural optimization, because some resulting quantities change significantly even after tiny changes of geometry or material parameters. Another undesirable feature is sensitivity to numerical options of the models, such as contact algorithm, contact penalty stiffness and time-step. The current contribution is aiming to show that buckling under contact constraints may be a source of such sensitivities. The influence of contact formulation, contact stiffness and initial geometric imperfections are also presented.

Crashworthiness of a car depends significantly on the crushing pattern of front rail, which includes buckling, self-contact and contact with other members. But it is difficult to trace origins of scatter even in crushing of a single rail. Instead the basic features of buckling under contact constraints are demonstrated on a simple setup (see Figure 1). An imperfect clamped beam is subjected to axial force against smooth obstacle. It snaps back from the obstacle under some critical load  $N_{cr}$ . A high sensitivity of the load-path to geometric imperfections (*even tiny*) is observed both in experiments and simulation. Different buckling patterns are obtained, which corresponds to quite different critical load (points  $a_3$  and  $s_3$ ).

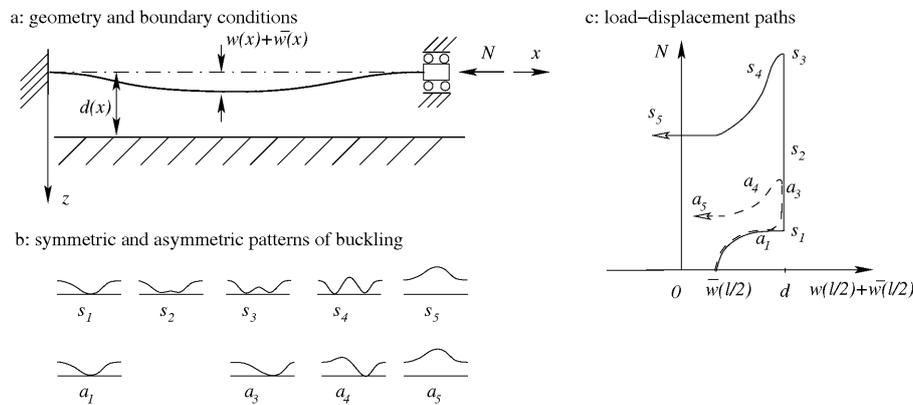


Figure 1: Contact buckling setup [1].

Numerical studies of the same model using FEA exhibited substantial dependence of the critical load on penalty stiffness both in quasi-static and dynamic simulations (see Table 1). Moreover, usage of pure Lagrange formulation over predicts the real critical load. A similar phenomenon for

another contact buckling setup was reported in [2]. Such sensitivity in case of crash simulation may also cause different crushing patterns, thus leading to scatter in intrusion, specific energy absorption and other quantities.

	Penalty factor	0.1	1.0	4.0	10.0	Lagrange
$N_{cr}/N_0$	Quasi-static ( $s_3$ )	0.267	0.310	0.356	>0.5	>2.0
	Dynamic ( $a_3$ )	0.151	0.149	-	0.125	0.125

Table 1: Dependency of the critical load on contact penalty factor and formulation.

The reason for this is non-convexity of the potential energy functional close to points  $(a_3, s_2-s_3)$ . Corresponding multiple equilibrium paths are close to each other and they create basins of attraction for corresponding dynamical processes. Therefore, it is sensible to perform analysis of how large are those basins at points of static or dynamic equilibrium. One possibility to do it is solving a non-convex minimization problem

$$u^* = \arg \min_{\substack{\|u-u_n\| < \varepsilon \\ u \in K}} \Pi(u), \quad (1)$$

$$K = \{w(x) \in C^1 : w(x) < d(x) \forall x\}, \quad (2)$$

which means to find in a specified vicinity of the current solution  $u_n$  a state  $u^*$  with negative secant tangent along direction in admissible set  $K$

$$\langle K^{\text{sec}}(u^* - u_n), u^* - u_n \rangle = \langle u^* - u_n, \partial\Pi(u^*) - \partial\Pi(u_n) \rangle. \quad (3)$$

If  $u_n$  is not equal to  $u^*$  then a basin is small and high scatter around the solution is expected. This is demonstrated on example of simple setup above and other related to crash simulation benchmarks.

#### References

- [1] Adan, N., Sheinman, I. and Altus E., "Post-buckling Behavior of Beams Under Contact Constraints," *J. Appl. Mech.*, **61**, 764-772 (1994).
- [2] Izi R., Schweizerhof K. and Konyukhov A. "Stability of thin walled structures strongly coupled with contact," in *Proc. Appl. Math. Mech.* (GAMM Conference Proceedings vol. **9**, Weinheim, 2009), Gdansk, Poland, February 9-13, 2009, 257-258 (2009).